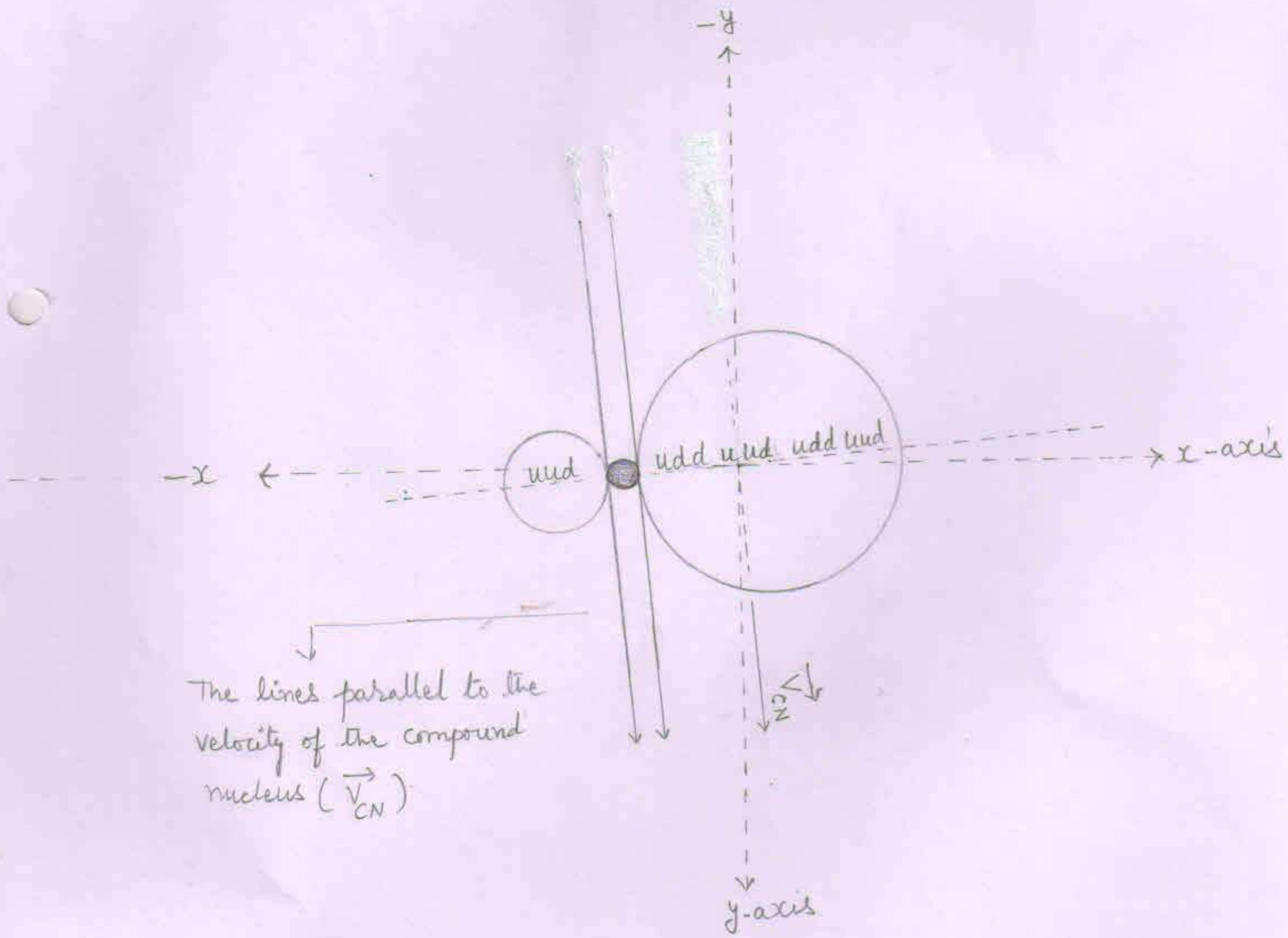
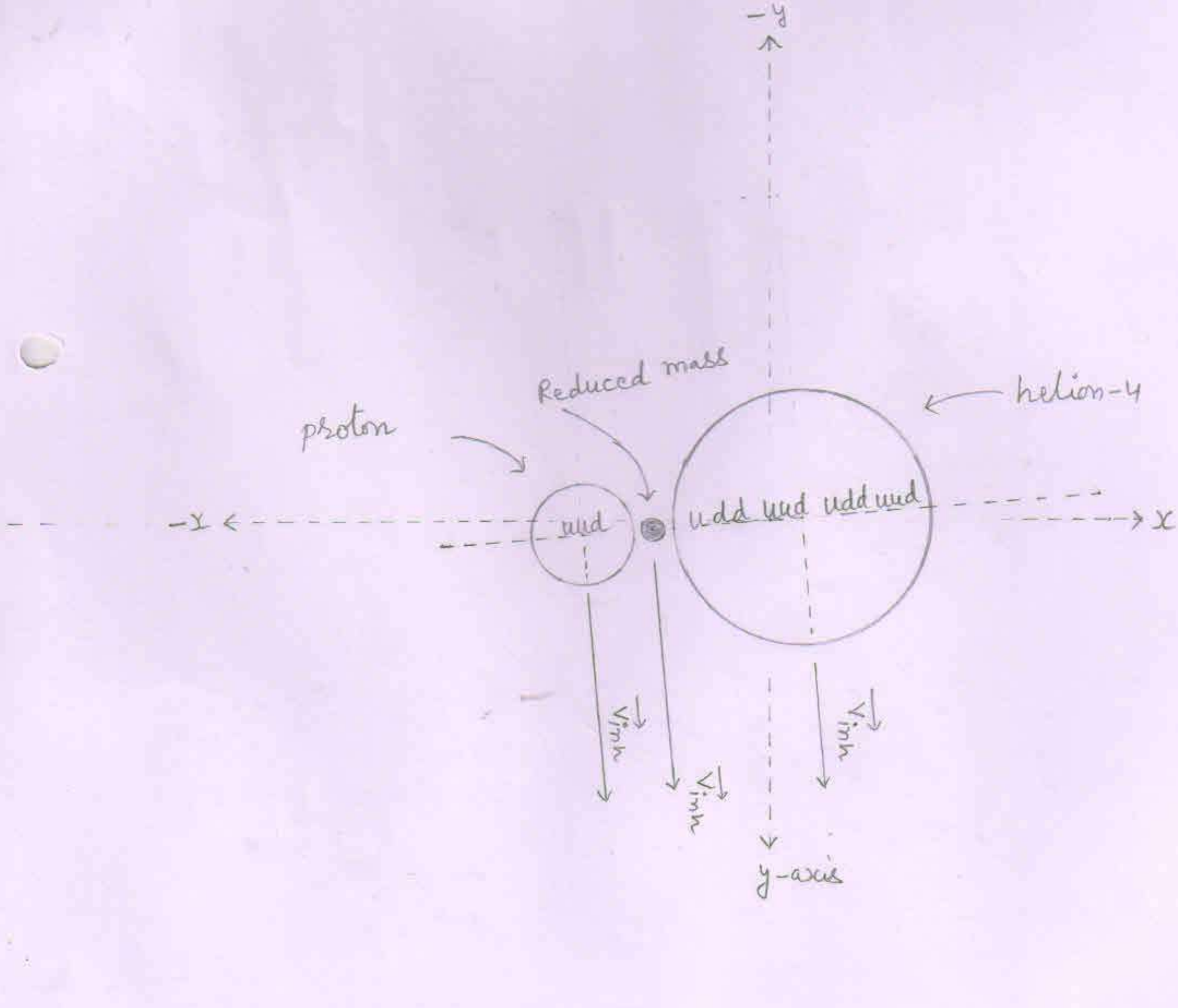
☐ The heterogenous compound nucleus to show the lines parallel to the $\vec{V}_{CN}$



The splitting of the heterogenous compound nucleus



Inhereted velocity of the particle(s)

⇒ Each particle has an inhereted velocity ($\vec{v}_{inh}$) equal to the velocity of compound nucleus ($\vec{v}_{CN}$)

I. Inhereted velocity of the particle ${}_2He^4$

1. $\vec{v}_{inh} = \vec{v}_{CN} = 0.5258 \times 10^7$ m/s

⇒ Components of inhereted velocity of the particle ${}_2He^4$

1. $\vec{v}_x = v_{inh} \cos\alpha = v_{CN} \cos\alpha = 0.0408 \times 10^7$ m/s

2. $\vec{v}_y = v_{inh} \cos\beta = v_{CN} \cos\beta = 0.5243 \times 10^7$ m/s

3. $\vec{v}_z = v_{inh} \cos\gamma = v_{CN} \cos\gamma = 0$ m/s

II. Inhereted velocity of the proton

1. $\vec{v}_{inh} = \vec{v}_{CN} = 0.5258 \times 10^7$ m/s

⇒ Components of inhereted velocity of the proton

1. $\vec{v}_x = v_{inh} \cos\alpha = v_{CN} \cos\alpha = 0.0408 \times 10^7$ m/s

2. $\vec{v}_y = v_{inh} \cos\beta = v_{CN} \cos\beta = 0.5243 \times 10^7$ m/s

3. $\vec{v}_z = v_{inh} \cos\gamma = v_{CN} \cos\gamma = 0$ m/s

III. Inhereted velocity of the reduced mass

Propellation of particles

Reduced mass converts into energy and the total energy (E_T) propell both the particles with equal and opposite momentum.

1. Reduced mass

$\Delta m = [m_d + m_{He-3}] - [m_p + m_{He-4}]$

$= [2.01355 + 3.0149] - [1.0072 + 4.0015]$ amu

$= [5.02845] - [5.0087]$ amu

$= 0.01975$ amu

$= 0.01975 \times 1.6605 \times 10^{-27}$ kg

$= 0.0327 \times 10^{-27}$ kg

2. Inhereted kinetic energy of reduced mass:

$E_{inh} = \frac{1}{2} \Delta m \, V_{CN}^2$

$= \frac{1}{2} \times 0.0327 \times 10^{-27} \times 0.27655513 \times 10^{14}$ J

$= 0.004521 \times 10^{-13}$ J

$= 0.002825$ Mev

3. Released energy

$$E_R = \Delta m c^2$$
$$= 0.01975 \times 931 \text{ Mev}$$
$$= 18.38725 \text{ Mev}$$

4. Total energy

$$E_T = E_{inh} + E_R$$

$$= [0.002825 + 18.38725] \text{ Mev}$$

$$= 18.390075 \text{ Mev}$$

116

Increament in the energy of the particle(s)

The total energy E_T is divided between the particles according to their inverse masses. so, the increased energy (E_{inc}) of the particle is –

1. For He-4

$$E_{inc} = \frac{m_p}{m_p + m_{He-4}} \times E_T$$

$$= \frac{1.0072 \text{ amu}}{[1.0072 + 4.0015] \text{ amu}} \times 18.390075 \text{ Mev}$$

$$= \frac{1.0072}{5.0087} \times 18.390075 \text{ Mev}$$

$$= 0.2010901 0322 \times 18.390075 \text{ Mev}$$

$$= 3.6980 \text{ Mev}$$

2. Increased energy of the proton

$$E_{inc} = [E_T] - [\text{Increased energy of He-4}]$$

$$= 18.390075 - 3.698 \text{ Mev}$$

$$= 14.692075 \text{ Mev}$$

Increased velocity of the particle

1. For proton

$$E_{inc} = \frac{1}{2} m_p v_{inc}^2$$

$$v_{inc} = \left[\frac{2 \times E_{inc}}{m_p} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 \times 14.692075 \times 1.6 \times 10^{-13}\ \mathrm{J}}{1.6726 \times 10^{-27}\ \mathrm{kg}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{47.01464 \times 10^{-13}}{1.6726 \times 10^{-27}} \right]^{\frac{1}{2}} \ \mathrm{m/s}$$

$$= \left[28.1087169675 \times 10^{14} \right]^{\frac{1}{2}} \ \mathrm{m/s}$$

$$= 5.3017 \times 10^{7} \ \mathrm{m/s}$$

2. For Helium-4

$$v_{inc} = \left[\frac{2 \times E_{inc}}{m_{He\text{-}4}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 \times 3.6980 \times 1.6 \times 10^{-13}\ \mathrm{J}}{6.64449 \times 10^{-27}\ \mathrm{kg}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{11.8336 \times 10^{-13}}{6.64449 \times 10^{-27}} \right]^{\frac{1}{2}} \ \mathrm{m/s}$$

= [1.7[illegible] $\times 10^{14}$]$^{\frac{1}{2}}$ m/s

Angle of propellation

1. As the reduced mass converts into energy, the total energy (E_T) propell both the particles with equal and opposite momentum according to a ray (line) perpendicular to the direction (line) of the velocity of the compound nucleus ($\vec{V}_{CN}$).

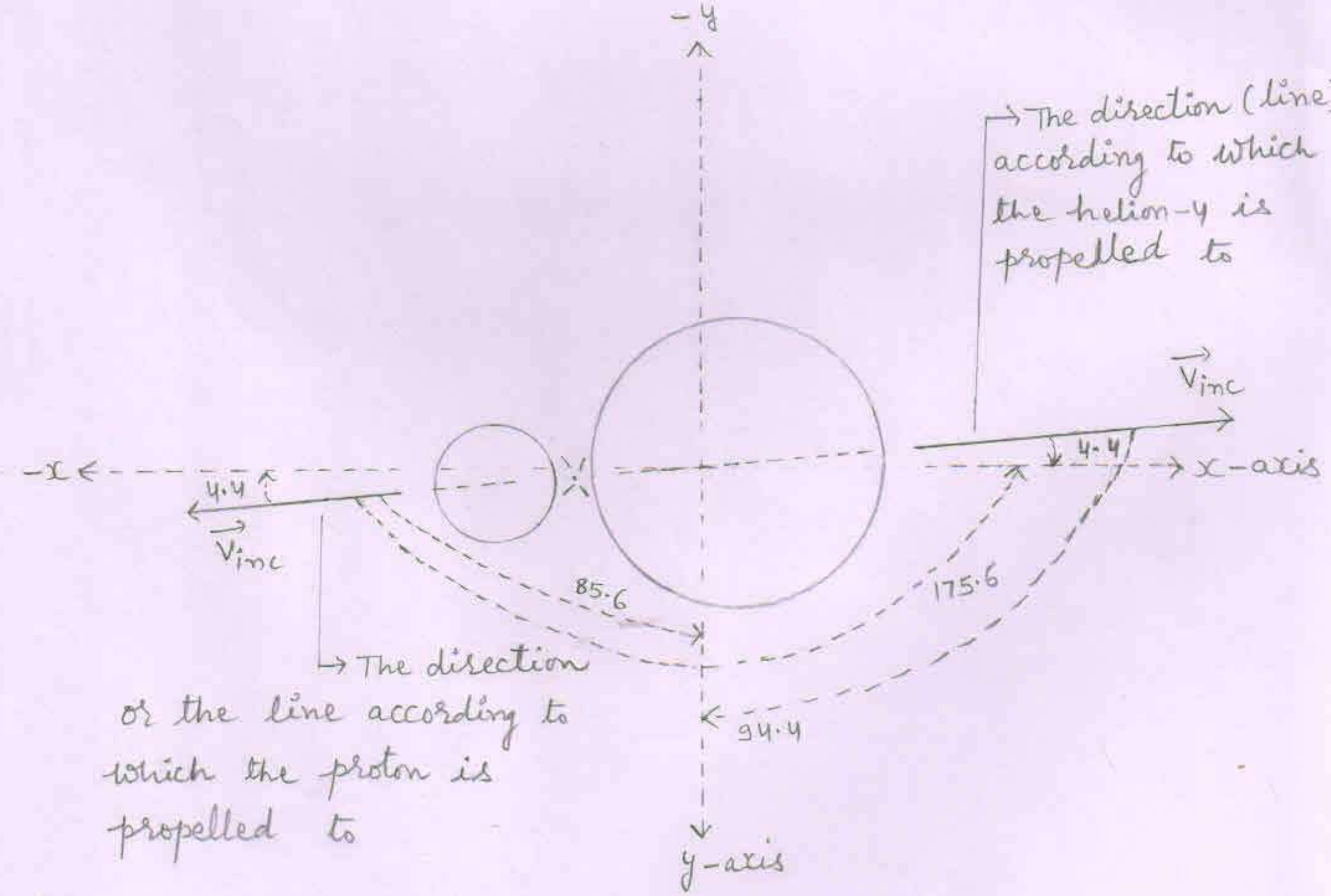
2. We know that when there a fusion process occures, then we find the lighter nucleus in the forward direction [or in the direction of ion beam or in the direction of the velocity of the compound nucleus ($\vec{V}_{CN}$).]

3. At point `F', as $\vec{V}_{CN}$ makes 85.6 degree angle with x-axis, 4.4 degree angle with y-axis and 90° angle with z-axis.

4. So, the proton is propelled making 175.6 degree angle with x-axis, 85.6 degree angle with y-axis and 90° angle with z-axis.

5. While the helion-4 is propelled making 4.4 degree angle with x-axis, 94.4 degree angle with y-axis and 90° angle with z-axis.

Propellation of the particles



components of the increased velocity ($\vec{v}_{inc}$) of the particles

I For proton

1. $\vec{v}_x = v_{inc} \cos\alpha$

$v_{inc} = 5.3017 \times 10^7$ m/s

$\cos\alpha = \cos(175.6) = -\cos(4.4) = -0.99$

$\Rightarrow \vec{v}_x = 5.3017 \times 10^7 \times (-0.99)$ m/s

$= -5.2486 \times 10^7$ m/s

2. $\vec{v}_y = v_{inc} \cos\beta$

$\cos\beta = \cos(85.6) = 0.07$

$\Rightarrow \vec{v}_y = 5.3017 \times 10^7 \times 0.07$ m/s

$= 0.3711 \times 10^7$ m/s

3. $\vec{v}_z = v_{inc} \cos\gamma$

$\cos\gamma = \cos 90^\circ = 0$

$\Rightarrow \vec{v}_z = 5.3017 \times 10^7 \times 0$ m/s

$= 0$ m/s

II For helion-4

1. $\vec{v}_x = v_{inc} \cos\alpha$

$v_{inc} = 1.3345 \times 10^7$ m/s

$\cos\alpha = \cos(4.4) = 0.99$

$\Rightarrow \vec{v}_x = 1.3345 \times 10^7 \times 0.99$ m/s

$= 1.3211 \times 10^7$ m/s

2. $\vec{v}_y = v_{inc} \cos\beta$

$\cos\beta = \cos(94.4) = -\cos(85.6) = -0.07$

$\Rightarrow \vec{v}_y = 1.3345 \times 10^7 \times (-0.07)$ m/s

$= -0.0934 \times 10^7$ m/s

3. $\vec{v}_z = v_{inc} \cos\gamma$

$\cos\gamma = \cos 90^\circ = 0$

$\Rightarrow \vec{v}_z = 1.3345 \times 10^7 \times 0$ m/s $= 0$ m/s components of the final velocity ($\vec{v}_f$) of the particles

I. For proton

According to	Inhereted velocity ($\vec{v}_{inh}$)	Increased velocity ($\vec{v}_{inc}$)	Final velocity ($\vec{v}_f$) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = 0.0408 \times 10^7$ m/s	$\vec{v}_x = -5.2486 \times 10^7$ m/s	$\vec{v}_x = -5.2078 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.5243 \times 10^7$ m/s	$\vec{v}_y = 0.3711 \times 10^7$ m/s	$\vec{v}_y = 0.8954 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

II For helion-4

According to	Inhereted velocity ($\vec{v}_{inh}$)	Increased velocity ($\vec{v}_{inc}$)	Final velocity ($\vec{v}_f$) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = 0.0408 \times 10^7$ m/s	$\vec{v}_x = 1.3211 \times 10^7$ m/s	$\vec{v}_x = 1.3619 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.5243 \times 10^7$ m/s	$\vec{v}_y = -0.0934 \times 10^7$ m/s	$\vec{v}_y = 0.4309 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

Final velocity (v_f) of the proton

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$v_x = 5.2078 \times 10^7$ m/s
$v_y = 0.8954 \times 10^7$ m/s
$v_z = 0$ m/s

$$\Rightarrow v_f^2 = (5.2078 \times 10^7)^2 + (0.8954 \times 10^7)^2 + (0)^2 \ m^2/s^2$$

$$\Rightarrow v_f^2 = (27.12118084 \times 10^{14}) + (0.80174116 \times 10^{14}) + 0 \ m^2/s^2$$

$$\Rightarrow v_f^2 = 27.922922 \times 10^{14} \ m^2/s^2$$

$$\Rightarrow v_f = 5.2842 \times 10^{14} \ m/s$$

Final kinetic energy of the proton

$$E = \frac{1}{2} m_p v_f^2$$

$v_f^2 = 27.922922 \times 10^{14}$ m²/s²

$$\Rightarrow E = \frac{1}{2} \times 1.6726 \times 10^{-27} \times 27.922922 \times 10^{14} \ J$$

$$= 23.3519396686 \times 10^{-13} \ J$$

$$= 14.5949 \ MeV$$

$$m_p v_f^2 = 1.6726 \times 10^{-27} \times 27.922922 \times 10^{14} \ J$$

$$= 46.7038 \times 10^{-13} \ J$$

Final velocity of the helion-4

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$v_x = 1.3619 \times 10^7$ m/s
$v_y = 0.4309 \times 10^7$ m/s
$v_z = 0$ m/s

$$\Rightarrow v_f^2 = (1.3619 \times 10^7)^2 + (0.4309 \times 10^7)^2 + (0)^2 \quad m^2/s^2$$

$$\Rightarrow v_f^2 = (1.85477161 \times 10^{14}) + (0.18567481 \times 10^{14}) + 0 \quad m^2/s^2$$

$$\Rightarrow v_f^2 = 2.04044642 \times 10^{14} \quad m^2/s^2$$

$$\Rightarrow v_f = 1.4284 \times 10^7 \text{ m/s}$$

Final kinetic energy of the helion-4

$$E = \frac{1}{2} m_{He-4} v_f^2$$

$v_f^2 = 2.04044642 \times 10^{14}$ m^2/s^2

$$\Rightarrow E = \frac{1}{2} \times 6.64449 \times 10^{-27} \times 2.04044642 \times 10^{14} \text{ J}$$

$$= 6.7788629166 \times 10^{-13} \text{ J}$$
$$= 4.2367 \text{ MeV}$$

$$m_{He-4} v_f^2 = 6.64449 \times 10^{-27} \times 2.04044642 \times 10^{14} \text{ J}$$
$$= 13.5577 \times 10^{-13} \text{ J}$$

Acting forces on the proton

1. $F_y = q V_x B_z \sin\theta$

$\vec{V_x} = -5.2078 \times 10^{7}$ m/s

$\vec{B_z} = -1$ Tesla

$q = 1.6 \times 10^{-19}$ C

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_y = 1.6 \times 10^{-19} \times 5.2078 \times 10^{7} \times 1 \times 1$ N

$= 8.3324 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F_y}$ is according to $+y$ axis. So,

$$\vec{F_y} = 8.3324 \times 10^{-12} \text{ N}$$

2. $F_z = q V_x B_y \sin\theta$

$\vec{B_y} = 1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_z = 1.6 \times 10^{-19} \times 5.2078 \times 10^{7} \times 1 \times 1$ N

$= 8.3324 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F_z}$ is according to $+z$ axis. So,

$$\vec{F_z} = 8.3324 \times 10^{-12} \text{ N}$$

3. $F_x = q V_y B_z \sin\theta$

$\vec{V_y} = 0.8954 \times 10^{7}$ m/s

$\sin\theta = \sin 90^\circ = 1$

$F_x = 1.6 \times 10^{-19} \times 0.8954 \times 10^{7} \times 1 \times 1$ N

$= 1.4326 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F_x}$ is according to $+x$ axis.

4. Resultant force (F_R):

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$$F_x = 1.4326 \times 10^{-12} \text{ N}$$

$$F = F_y = F_z = 8.3324 \times 10^{-12} \text{ N}$$

$$\Rightarrow F_R^2 = F_x^2 + 2F^2$$

$$\Rightarrow F_R^2 = (1.4326 \times 10^{-12})^2 + 2(8.3324 \times 10^{-12})^2 \quad N^2$$

$$\Rightarrow F_R^2 = (2.05234276 \times 10^{-24}) + 2(69.42888976 \times 10^{-24}) \; N^2$$

$$\Rightarrow F_R^2 = (2.05234276 \times 10^{-24}) + (138.85777952 \times 10^{-24}) \; N^2$$

$$\Rightarrow F_R^2 = 140.91012228 \times 10^{-24} \quad N^2$$

$$\Rightarrow F_R = 11.8705 \times 10^{-12} \quad N$$

Radius of the circular orbit followed by the proton

$$r = \frac{mv^2}{F_R}$$

$mv^2 = 46.7038 \times 10^{-13}\ J$

$F_R = 11.8705 \times 10^{-12}\ N$

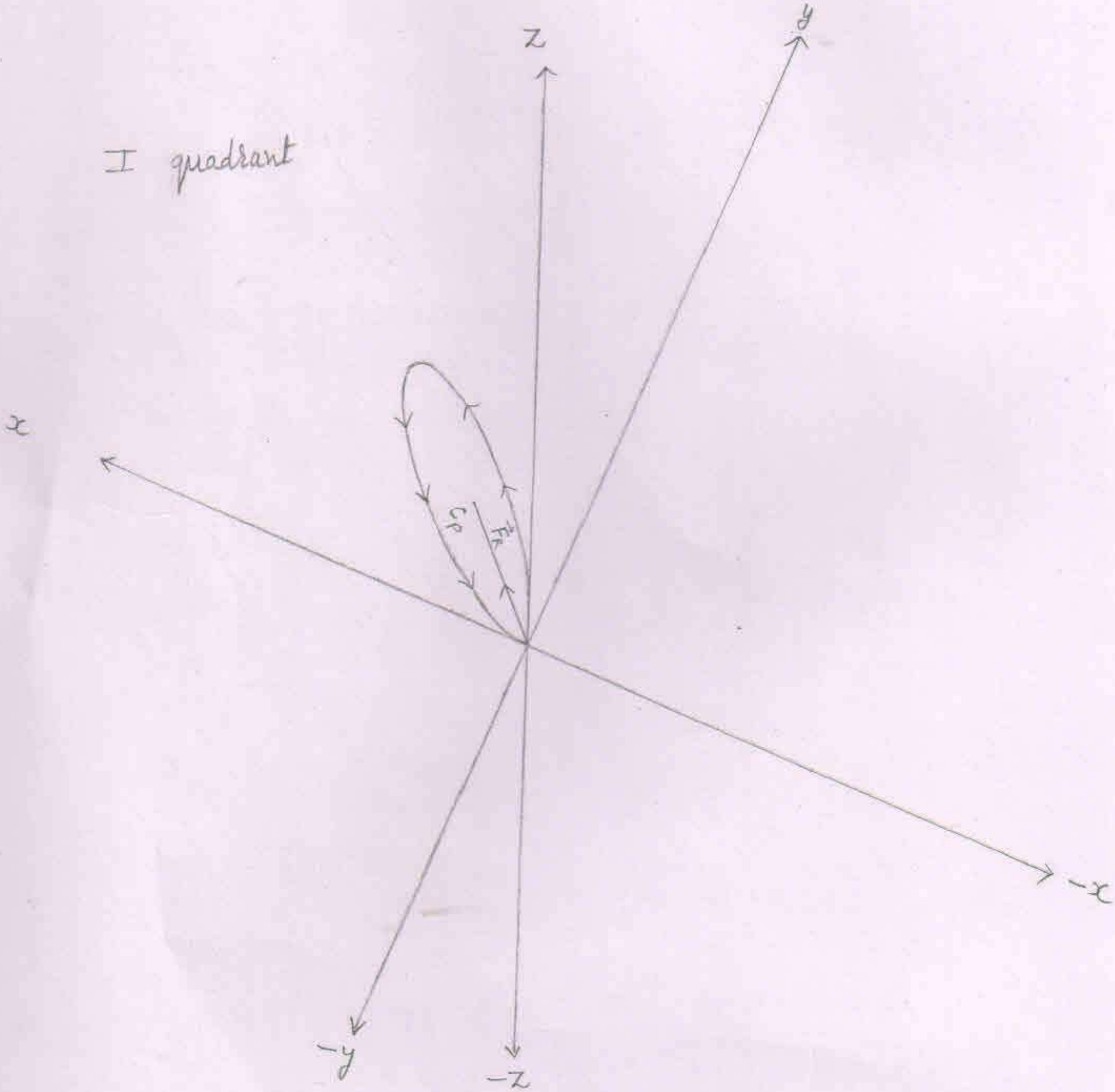
$$\Rightarrow r = \frac{46.7038 \times 10^{-13}}{11.8705 \times 10^{-12}} \ \frac{J}{N}$$

$$\Rightarrow r = 3.93444 \times 10^{-1}\ m$$

$$\Rightarrow r = 39.3444 \times 10^{-2}\ m$$

⇒ The circular orbit to be followed by the proton lies in the I (up) quadrant made up of positive x axis, positive y axis, and the positive z axis.

⇒ c_p = centre of the circle obtained by the proton.



Angles that make the resultant force (F_R) [acting on the proton when the proton is at point 'F'] with positive x, y and z-axes.

1. With x-axis

$$\cos\alpha = \frac{F_R \cos\alpha}{F_R} = \frac{\vec{F_x}}{F_R} = \frac{1.4326 \times 10^{-12}}{11.8705 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\alpha = 0.1206$

$\Rightarrow \alpha \simeq 83.1$ degree $[\because \cos(83.1) = 0.1201]$

2 With y-axis

$$\cos\beta = \frac{F_R \cos\beta}{F_R} = \frac{\vec{F_y}}{F_R} = \frac{8.3324 \times 10^{-12}}{11.8705 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\beta = 0.7019$

$\Rightarrow \beta \simeq 45.5$ degree $[\because \cos(45.5) = 0.7009]$

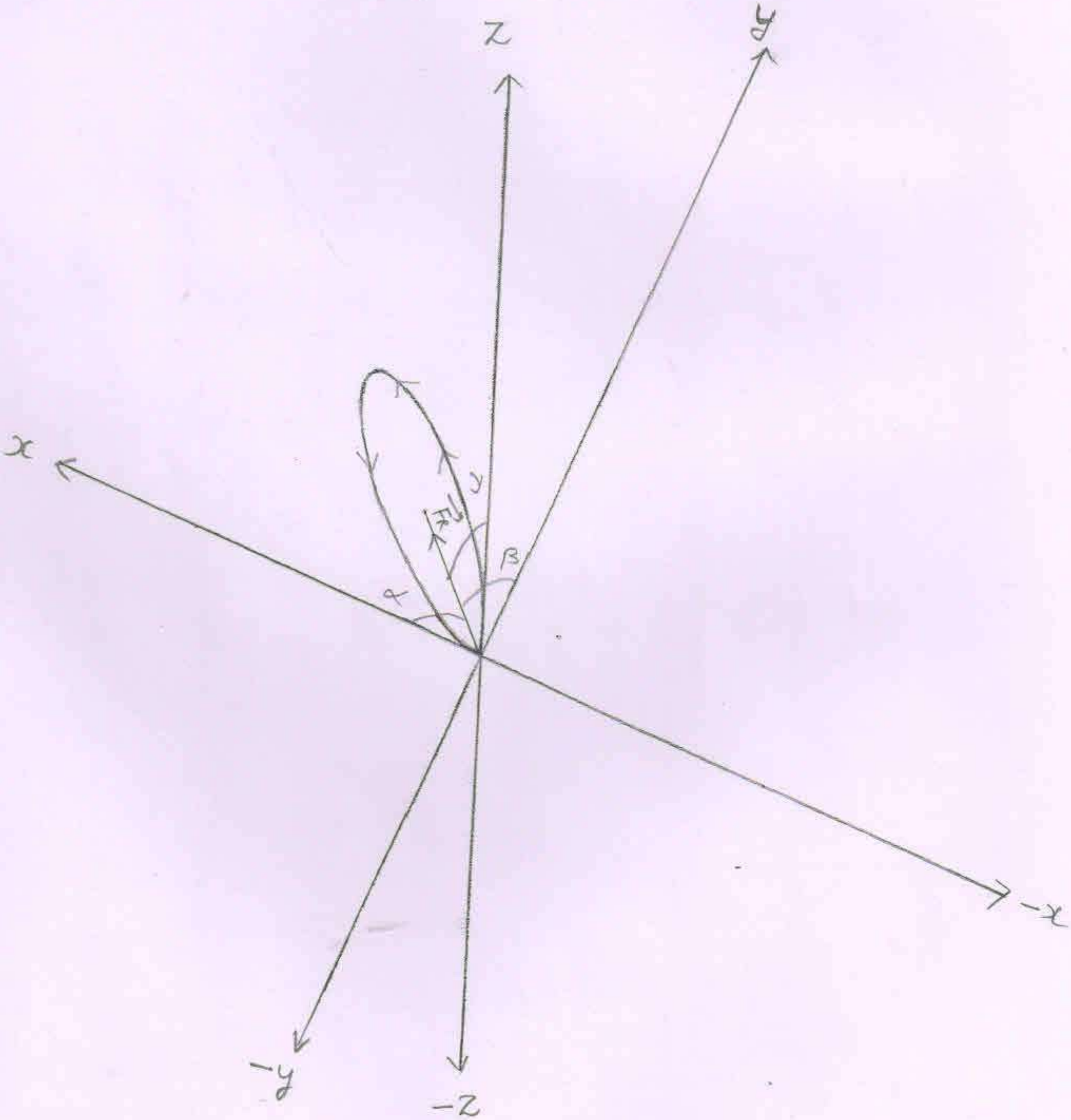
3. With z-axis

$$\cos\gamma = \frac{F_R \cos\gamma}{F_R} = \frac{\vec{F_z}}{F_R} = \frac{8.3324 \times 10^{-12}}{11.8705 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\gamma = 0.7019$

$\Rightarrow \gamma \simeq 45.5$ degree

Angles that make the resultant force ($\vec{F}_R$) at point F with the positive x, y and z axes.



Where,

$\alpha \simeq 83.1$

$\beta \simeq 45.5$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the proton

1. $\cos\alpha = \dfrac{x_2 - x_1}{d}$

$d = 2 \times r$
$= 2 \times 39.3444 \times 10^{-2}\,m$
$= 78.6888 \times 10^{-2}\,m$

$\cos\alpha = 0.12$

$\Rightarrow x_2 - x_1 = d \times \cos\alpha$
$\Rightarrow x_2 - x_1 = 78.6888 \times 10^{-2} \times 0.12\,m$
$\Rightarrow x_2 - x_1 = 9.4426 \times 10^{-2}\,m$
$\Rightarrow x_2 = 9.4426 \times 10^{-2}\,m \quad [\because x_1 = 0]$

2. $\cos\beta = \dfrac{y_2 - y_1}{d}$

$\cos\beta = 0.70$

$\Rightarrow y_2 - y_1 = d \times \cos\beta$
$\Rightarrow y_2 - y_1 = 78.6888 \times 10^{-2} \times 0.70\,m$
$\Rightarrow y_2 - y_1 = 55.0821 \times 10^{-2}\,m$
$\Rightarrow y_2 = 55.0821 \times 10^{-2}\,m \quad [\because y_1 = 0]$

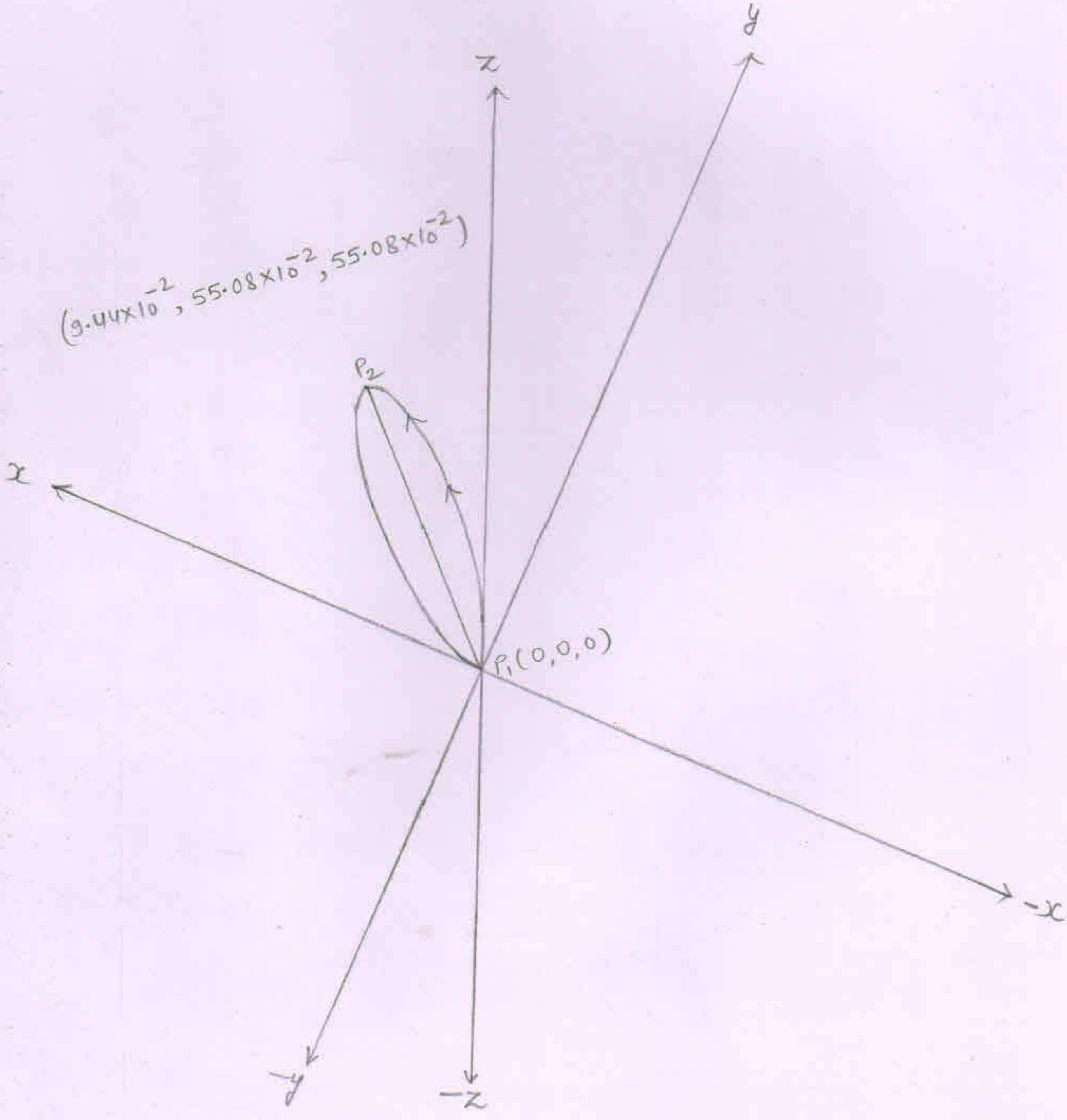
3. $\cos\gamma = \dfrac{z_2 - z_1}{d}$

$\cos\gamma = 0.70$

$\Rightarrow z_2 - z_1 = d \times \cos\gamma$
$\Rightarrow z_2 - z_1 = 78.6888 \times 10^{-2} \times 0.70\,m$
$\Rightarrow z_2 - z_1 = 55.0821 \times 10^{-2}\,m$
$\Rightarrow z_2 = 55.0821 \times 10^{-2}\,m \quad [\because z_1 = 0]$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the proton are as shown below.

$\Rightarrow$ The line $\overline{P_1 P_2}$ is the diameter of the circle.



Acting forces on the helion-4

1. $F_y = q V_x B_z \sin\theta$

$\vec{V}_x = 1.3619 \times 10^{7}$ m/s

$\vec{B}_z = -1$ Tesla

$q = 2 \times 1.6 \times 10^{-19}$ C

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_y = 2 \times 1.6 \times 10^{-19} \times 1.3619 \times 10^{7} \times 1 \times 1$ N

$= 4.3580 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_y$ is according to $-y$ axis. So,

$$\vec{F}_y = -4.3580 \times 10^{-12} \text{ N}$$

2. $F_z = q V_x B_y \sin\theta$

$\vec{B}_y = 1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_z = 2 \times 1.6 \times 10^{-19} \times 1.3619 \times 10^{7} \times 1 \times 1$ N

$= 4.3580 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_z$ is according to $-z$ axis. So,

$$\vec{F}_z = -4.3580 \times 10^{-12} \text{ N}$$

3. $F_x = q V_y B_z \sin\theta$

$\vec{V}_y = 0.4309 \times 10^{7}$ m/s

$\vec{B}_z = -1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_x = 2 \times 1.6 \times 10^{-19} \times 0.4309 \times 10^{7} \times 1 \times 1$ N

$= 1.3788 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_x$ is according to $+x$ axis. So,

$$\vec{F}_x = 1.3788 \times 10^{-12} \text{ N}$$

4. Resultant force (F_R) :

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$$F_x = 1.3788 \times 10^{-12}\ N$$

$$F = F_y = F_z = 4.3580 \times 10^{-12}\ N$$

$$\Rightarrow F_R^2 = F_x^2 + 2F^2$$

$$\Rightarrow F_R^2 = (1.3788 \times 10^{-12})^2 + 2\,(4.3580 \times 10^{-12})^2\ N^2$$

$$\Rightarrow F_R^2 = (1.90108944 \times 10^{-24}) + 2(18.992164 \times 10^{-24})\ N^2$$

$$\Rightarrow F_R^2 = (1.90108944 \times 10^{-24}) + (37.984328 \times 10^{-24})\ N^2$$

$$\Rightarrow F_R^2 = 39.88541744 \times 10^{-24}\ N^2$$

$$\Rightarrow F_R = 6.3154 \times 10^{-12}\ N$$

5. Radius of the circular orbit followed by the helion-4

$$r = \frac{mv^2}{F_R}$$

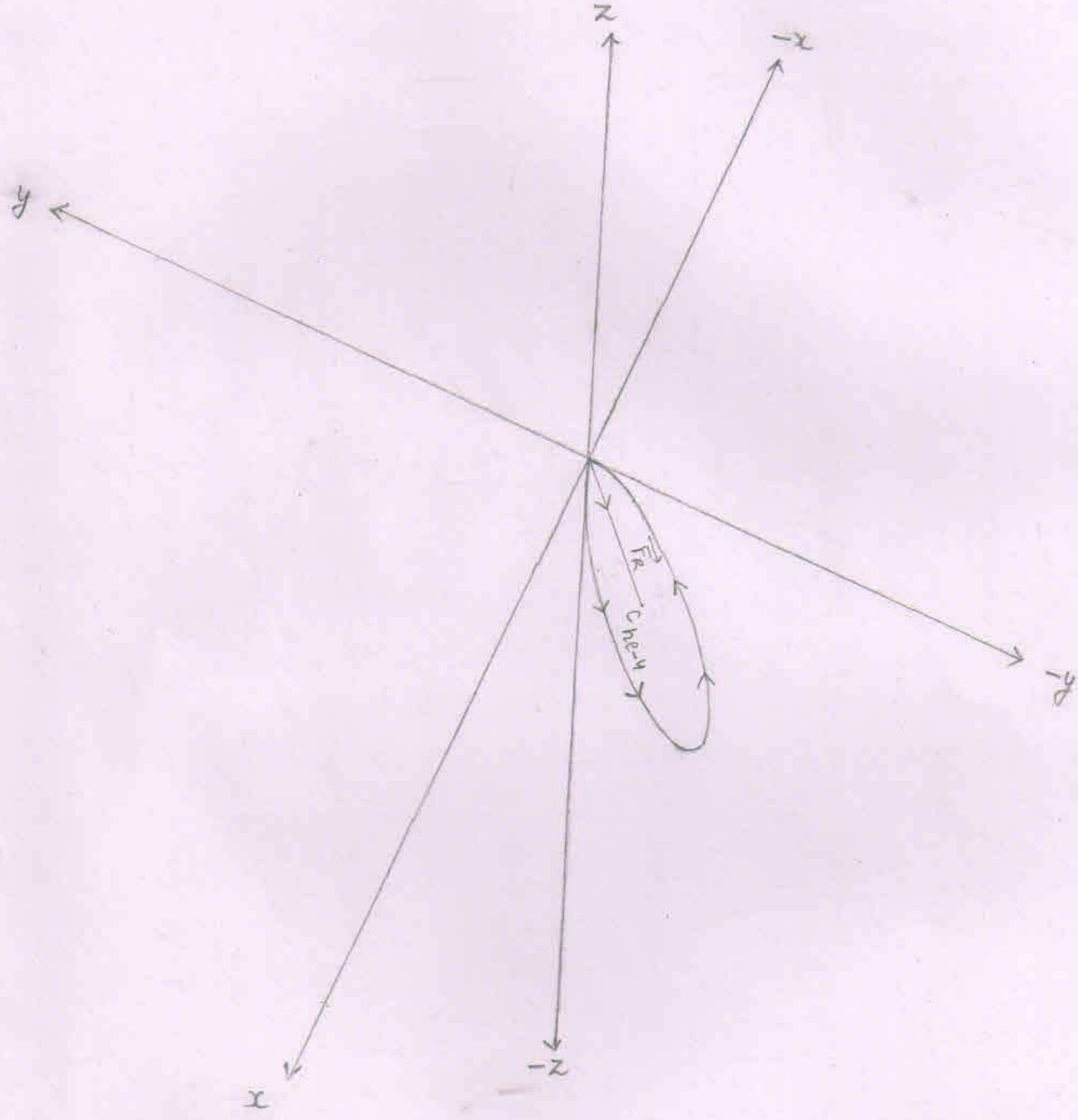
$mv^2 = 13.5577 \times 10^{-13}$ J

$F_R = 6.3154 \times 10^{-12}$ N

$$\Rightarrow r = \frac{13.5577 \times 10^{-13}}{6.3154 \times 10^{-12}} \frac{J}{N}$$

$$\Rightarrow r = 2.14676 \times 10^{-1} \text{ m}$$

$$\Rightarrow r = 21.4676 \times 10^{-2} \text{ m}$$



⇒ The circular orbit to be followed by the helion-4 lies in the IV (down) quadrant made up of positive x axis, negative y axis and the negative z axis.

⇒ C_{He-4} = centre of the circle to be followed by the helion-4

Angles that make the resultant force ($\vec{F}_R$) [acting on the helium-4 nucleus when the helium-4 is at point 'F'] with positive x, y and z-axes.

1. With x-axis

$$\cos\alpha = \frac{F_R \cos\alpha}{F_R} = \frac{\vec{F_x}}{F_R} = \frac{1.3788\times10^{-12}}{6.3154\times10^{-12}}\,\frac{N}{N}$$

$\Rightarrow \cos\alpha = 0.2183$

$\Rightarrow \alpha \simeq 77.4$ degree $[\because \cos(77.4) = 0.2181]$

2. With y-axis

$$\cos\beta = \frac{F_R \cos\beta}{F_R} = \frac{\vec{F_y}}{F_R} = \frac{-4.3580\times10^{-12}}{6.3154\times10^{-12}}\,\frac{N}{N}$$

$\Rightarrow \cos\beta = -0.6900$

$\Rightarrow \beta \simeq 133.6$ degree $[\because \cos(133.6) = -0.6896]$

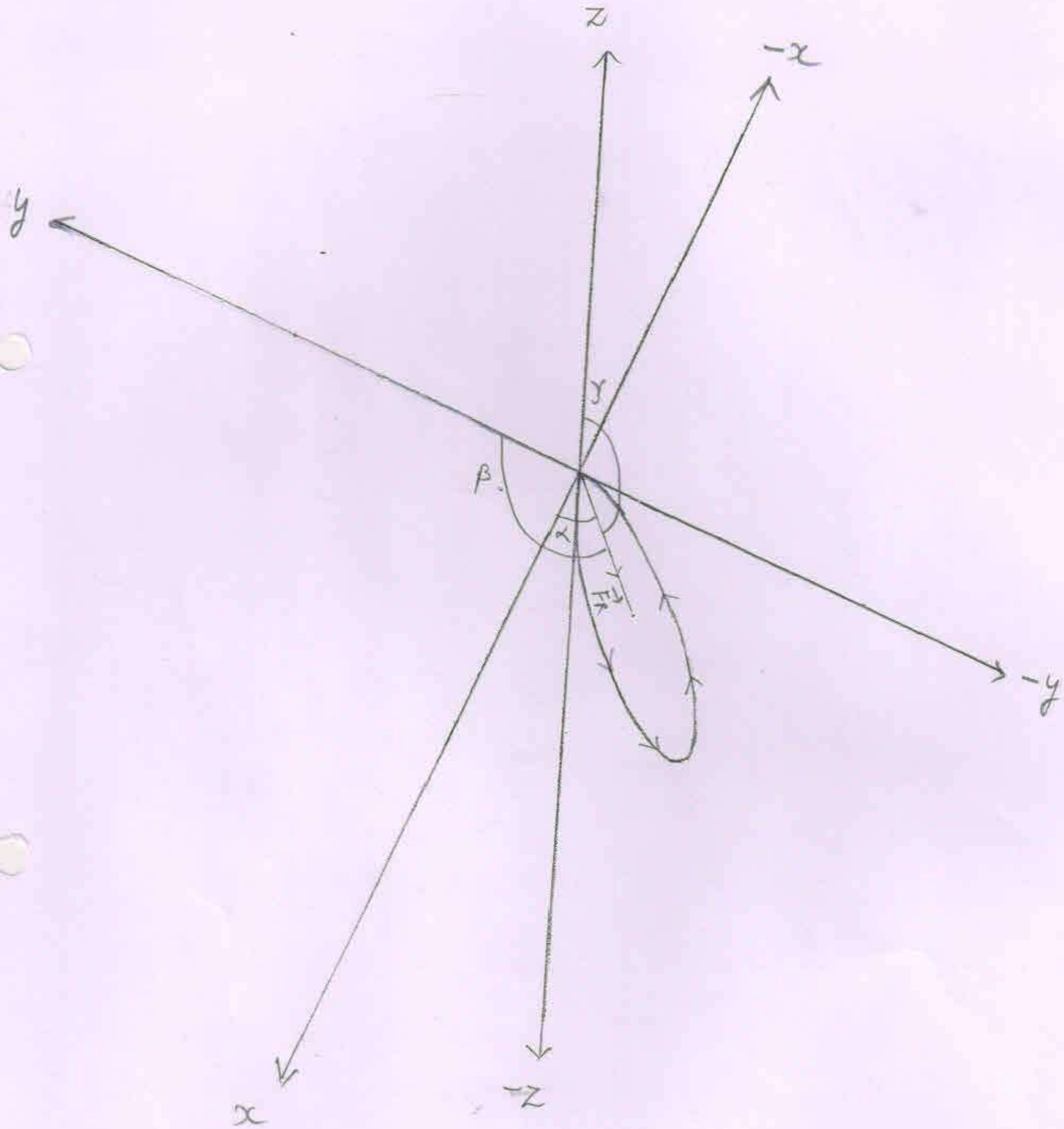
3. With z-axis

$$\cos\gamma = \frac{F_R \cos\gamma}{F_R} = \frac{\vec{F_z}}{F_R} = \frac{-4.3580\times10^{-12}}{6.3154\times10^{-12}}\,\frac{N}{N}$$

$\Rightarrow \cos\gamma = -0.6900$

$\Rightarrow \gamma \simeq 133.6$ degree

Angles that make the resultant force ($\vec{F_R}$) at point F with positive x, y and z axes.



Where, $\alpha \simeq 77.4$

$\beta \simeq 133.6$

$\gamma \simeq 133.6$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the helion-4

1. $\cos\alpha = \dfrac{x_2 - x_1}{d}$

$d = 2 \times r$

$= 2 \times 21.4676 \times 10^{-2}\,m$

$= 42.9352 \times 10^{-2}\,m$

$\cos\alpha = 0.21$

$\Rightarrow x_2 - x_1 = d \times \cos\alpha$

$\Rightarrow x_2 - x_1 = 42.9352 \times 10^{-2} \times 0.21\,m$

$\Rightarrow x_2 - x_1 = 9.0163 \times 10^{-2}\,m$

$\Rightarrow x_2 = 9.0163 \times 10^{-2}\,m \quad [\because x_1 = 0]$

2. $\cos\beta = \dfrac{y_2 - y_1}{d}$

$\cos\beta = -0.69$

$\Rightarrow y_2 - y_1 = d \times \cos\beta$

$\Rightarrow y_2 - y_1 = 42.9352 \times 10^{-2} \times (-0.69)\,m$

$\Rightarrow y_2 - y_1 = -29.6252 \times 10^{-2}\,m$

$\Rightarrow y_2 = -29.6252 \times 10^{-2}\,m \quad [\because y_1 = 0]$

3. $\cos\gamma = \dfrac{z_2 - z_1}{d}$

$\cos\gamma = -0.69$

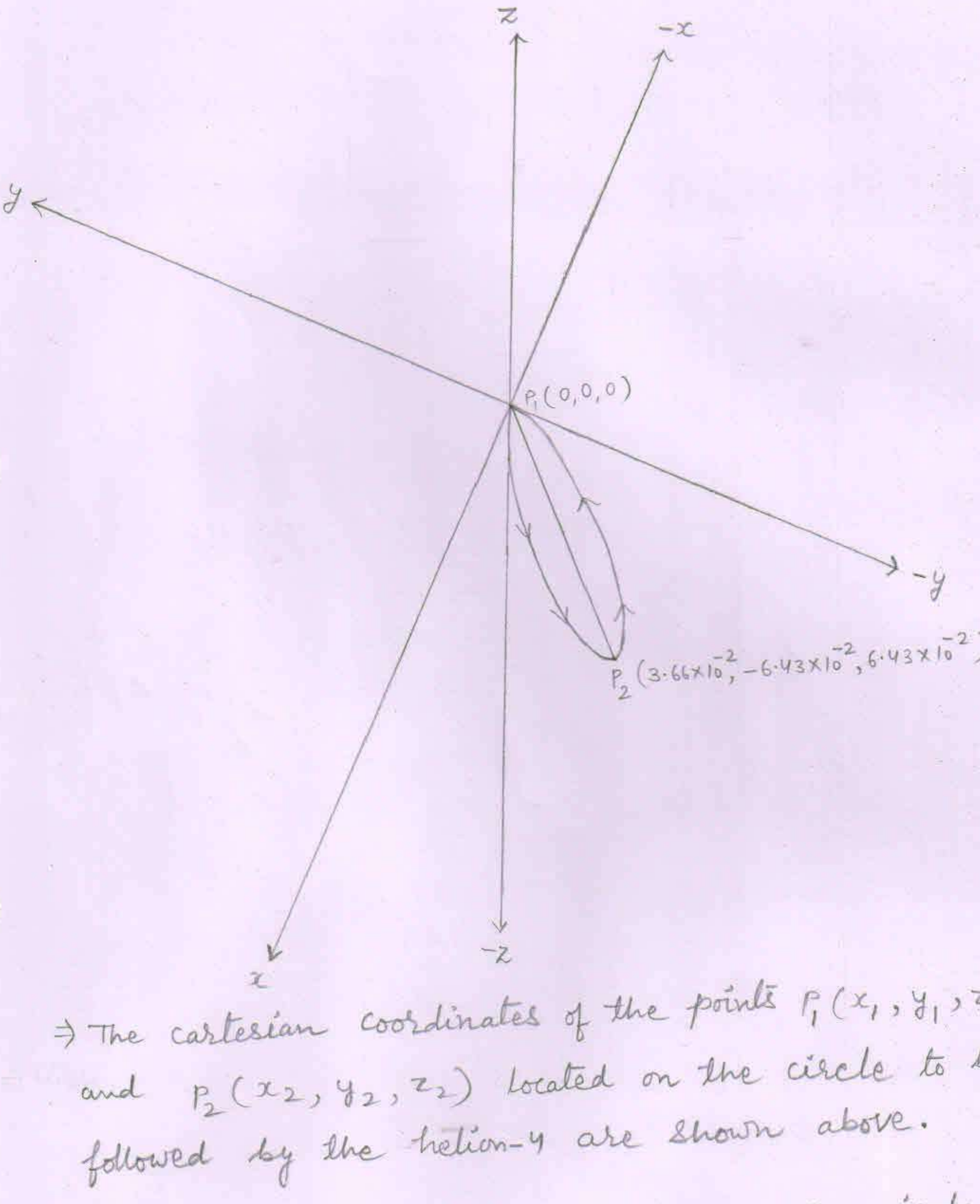
$\Rightarrow z_2 - z_1 = d \times \cos\gamma$

$\Rightarrow z_2 - z_1 = 42.9352 \times 10^{-2} \times (-0.69)\,m$

$\Rightarrow z_2 - z_1 = -29.6252 \times 10^{-2}\,m$

$\Rightarrow z_2 = -29.6252 \times 10^{-2}\,m \quad [\because z_1 = 0]$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$



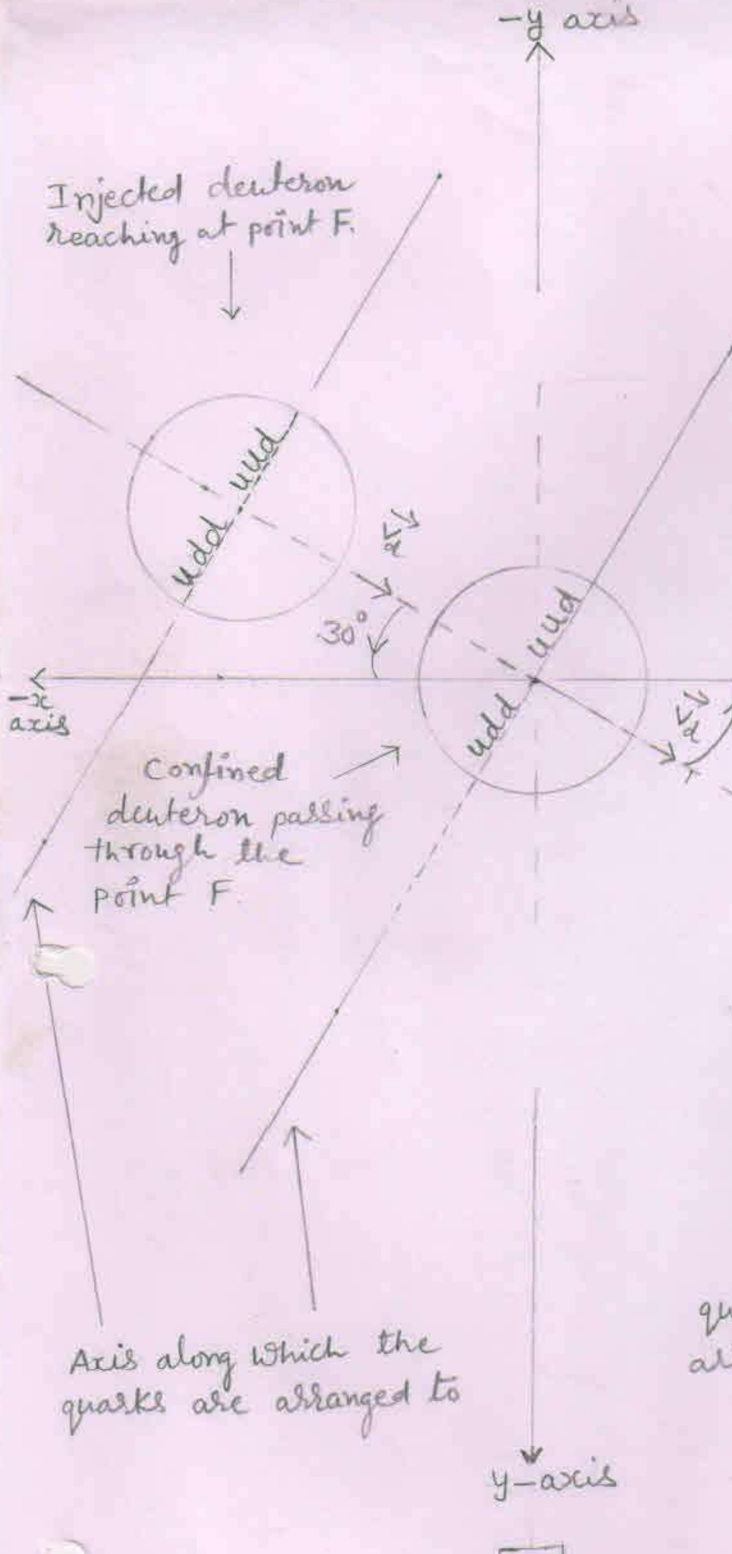
⇒ The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circle to be followed by the helion-4 are shown above.

⇒ The line $\overline{P_1P_2}$ is the diameter of the circle.

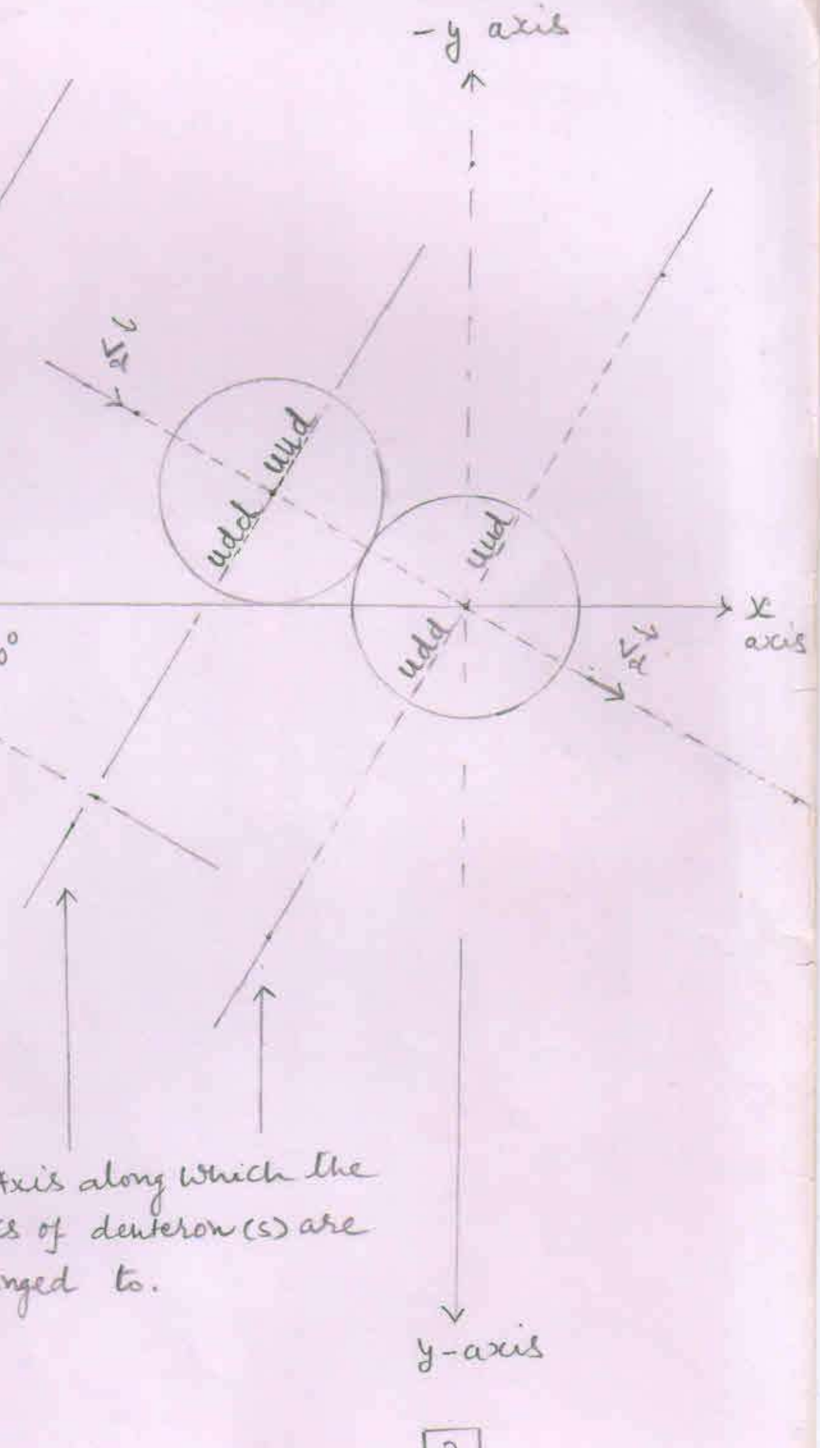
For fusion reaction : ${}^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{1}H + {}^{1}_{1}H$

1. Interaction of nuclei :-

The injected deuteron reaches at point 'F' and interacts [experiences a repulsive force due to confined deuteron] with the confined deuteron passing through the point 'F'. The injected deuteron overcomes the electrostatic repulsive force and - a like two solid spheres join - the injected deuteron dissimilarly joins with the confined deuteron.



Injection of the deuteron

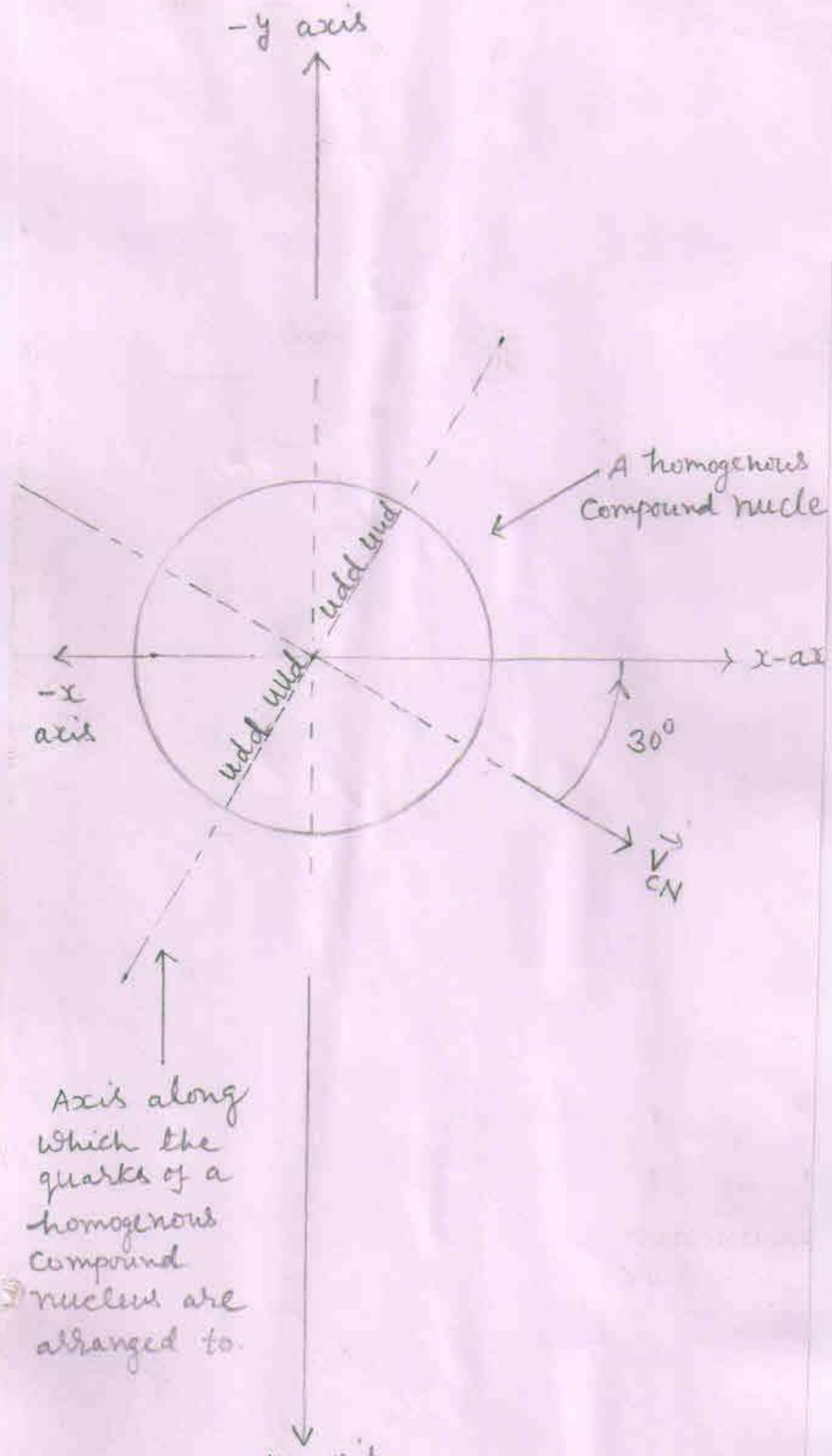


Interaction of nuclei

⇒

2. Formation of a homogenous compound nucleus :-
The constituents (quarks and gluons) of the dissimilarly joined nuclei (deuterons) behave like a liquid and form a homogenous compound nucleus having similarly distributed groups of quarks with similarly distributed surrounding gluons.

Thus in a homogenous compound nucleus - each group of quarks is surrounded by gluons in equal proportion. So, within a homogenous compound nucleus there are 4 groups of quarks with surrounding gluons.



$\boxed{3}$

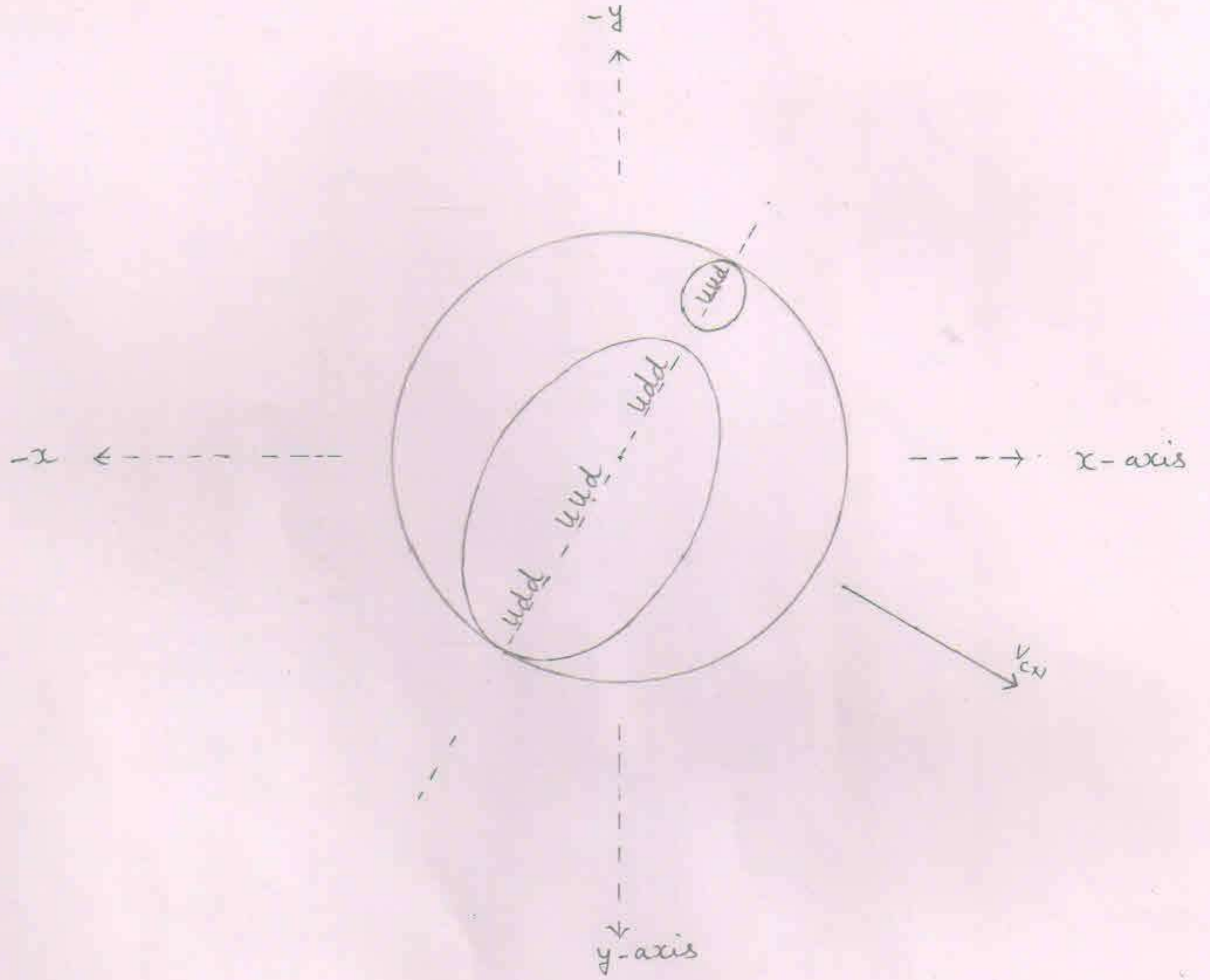
Formation of homogenous Compound nucleus

3. Formation of lobes within into the homogenous compound nucleus or the transformation of the homogenous compound nucleus into the heterogenous compound nucleus :-

The central group of quarks with its surrounding gluons to become a stable and the next higher nucleus (the triton) than the reactant one (the deuteron) includes the other two (nearby located) groups of quarks with their surrounding gluons and rearrange to form the 'A' lobe of the heterogenous compound nucleus.

While the remaining group of quarks to become a stable nucleus (the proton) includes its surrounding gluons or mass [out of the available mass (or gluons) that is not included in the formation of the lobe 'A'] and rearrange to form the 'B' lobe of the heterogenous compound nucleus.

Thus due to formation of two dissimilar lobes within into the homogenous compound nucleus, the homogenous compound nucleus transforms into the heterogenous compound nucleus.



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Formation of lobes

⇒ Within into the homogenous compound nucleus, the greater nucleus is the triton and the smaller one is the proton while the remaining space represents the remaining gluons.

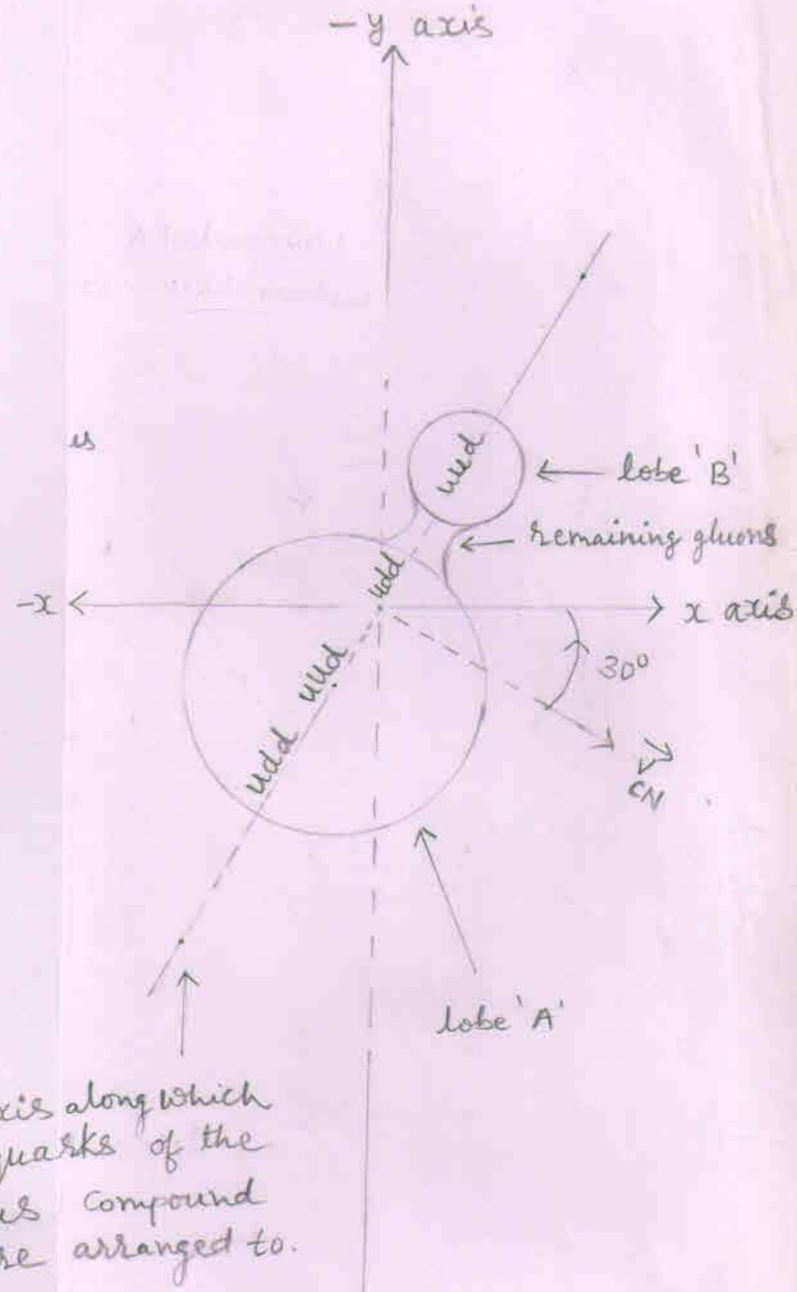
⇒ Within into the homogenous compound nucleus, the greater nucleus is the lobe 'A' while the smaller nucleus is the lobe 'B'.

4. Final stage of the heterogenous compound nucleus:-

The process of formation of lobes creates voids between the lobes. So, the remaining gluons [or the mass that is not involved in the formation of any lobe] rearrange to fill the void(s) between the lobes and thus the remaining gluons form a node between the two dissimilar lobes of the heterogenous compound nucleus.

Thus the reduced mass (or the remaining gluons) keeps both the dissimilar lobes - of the heterogenous compound nucleus - joined them together.

So, finally, the heterogenous compound nucleus becomes like an abnormal digit eight or becomes like a dumb-bell.



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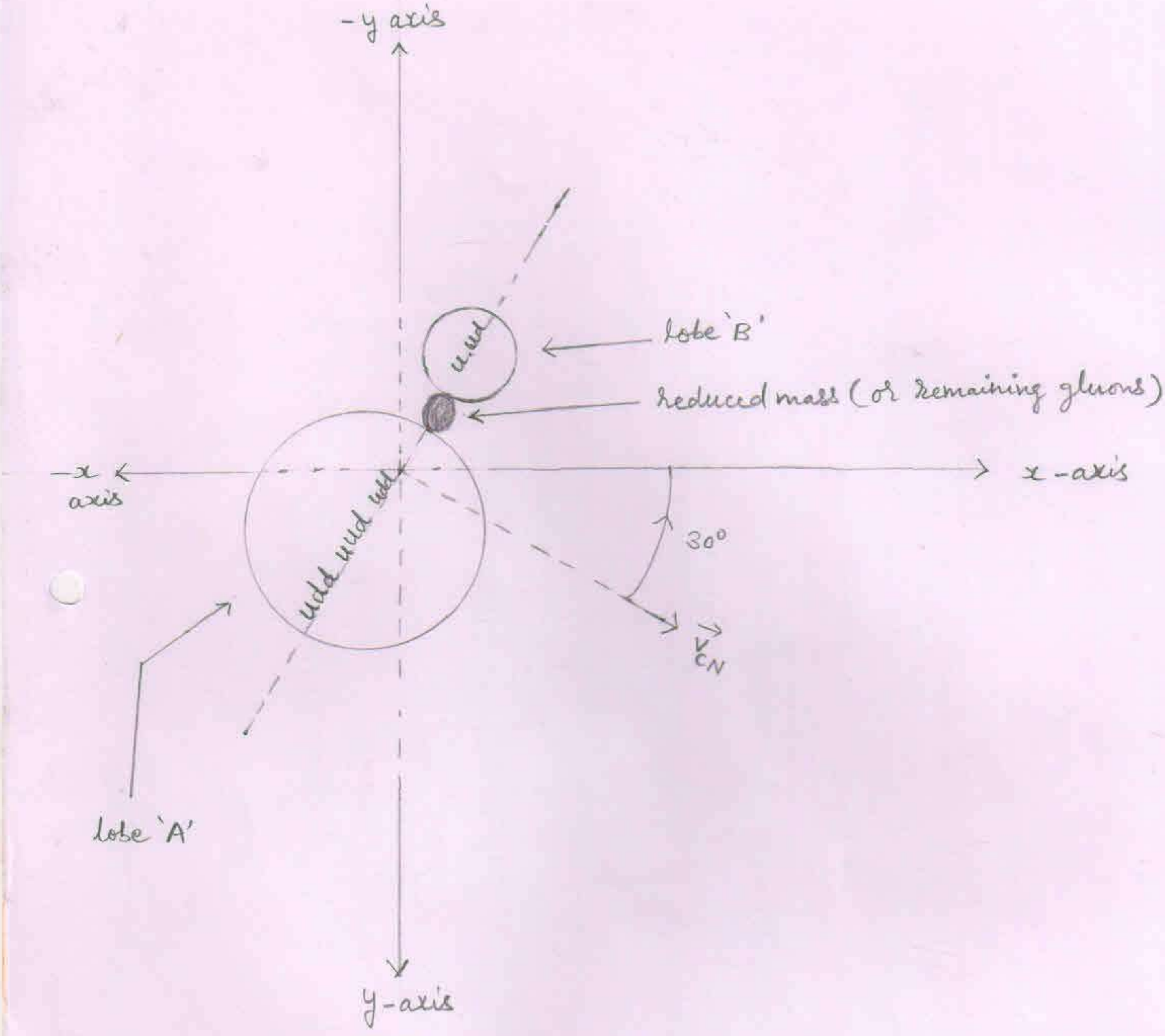
The heterogenous Compound nucleus



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Final Stage of a heterogenous Compound nucleus.

The splitting of the heterogenous compound nucleus :-

⇒ The heterogenous compound nucleus, due to its instability, splits according to the lines parallel to the direction of the velocity of the compound nucleus ($\vec{V}_{CN}$) into three particles - the triton, the proton and the reduced mass (Δm).

out of them, the two particles (the triton and the proton) are stable while the third one (reduced mass) is unstable.

⇒ According to the law of inertia, each particle that has separated from the compound nucleus, has an inhereted velocity ($\vec{V}_{inh}$) equal to the velocity of the compound nucleus ($\vec{V}_{CN}$).

⇒ So, for conservation of momentum

$$M\vec{V}_{CN} = (m_t + \Delta m + m_p)\,\vec{V}_{CN}$$

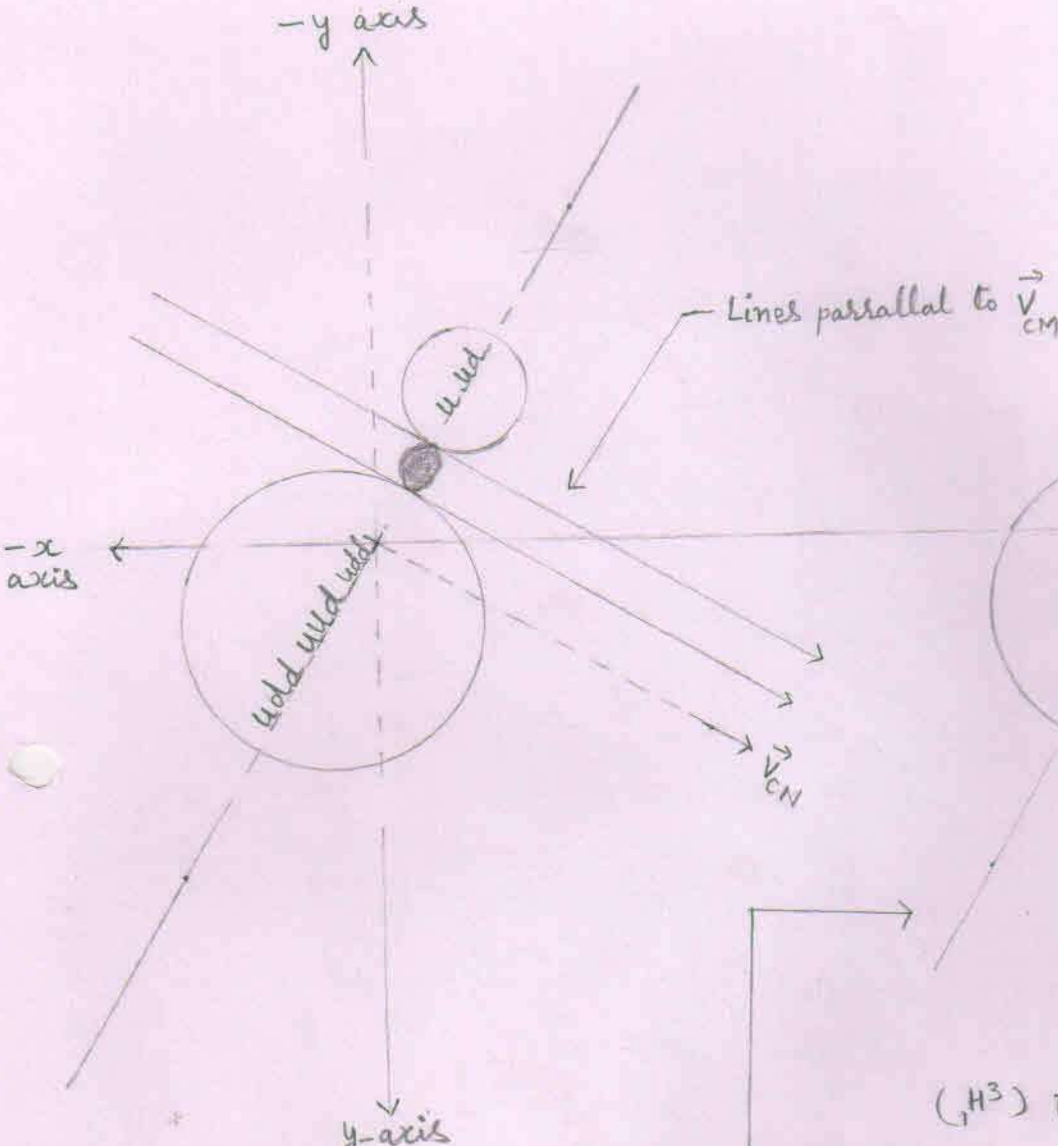
Where,

M = mass of the compound nucleus
$\vec{V}_{CN}$ = velocity of the compound nucleus

m_t = mass of the triton.

Δm = reduced mass
m_p = mass of the proton

The Splitting of a heterogehous compound nucleus



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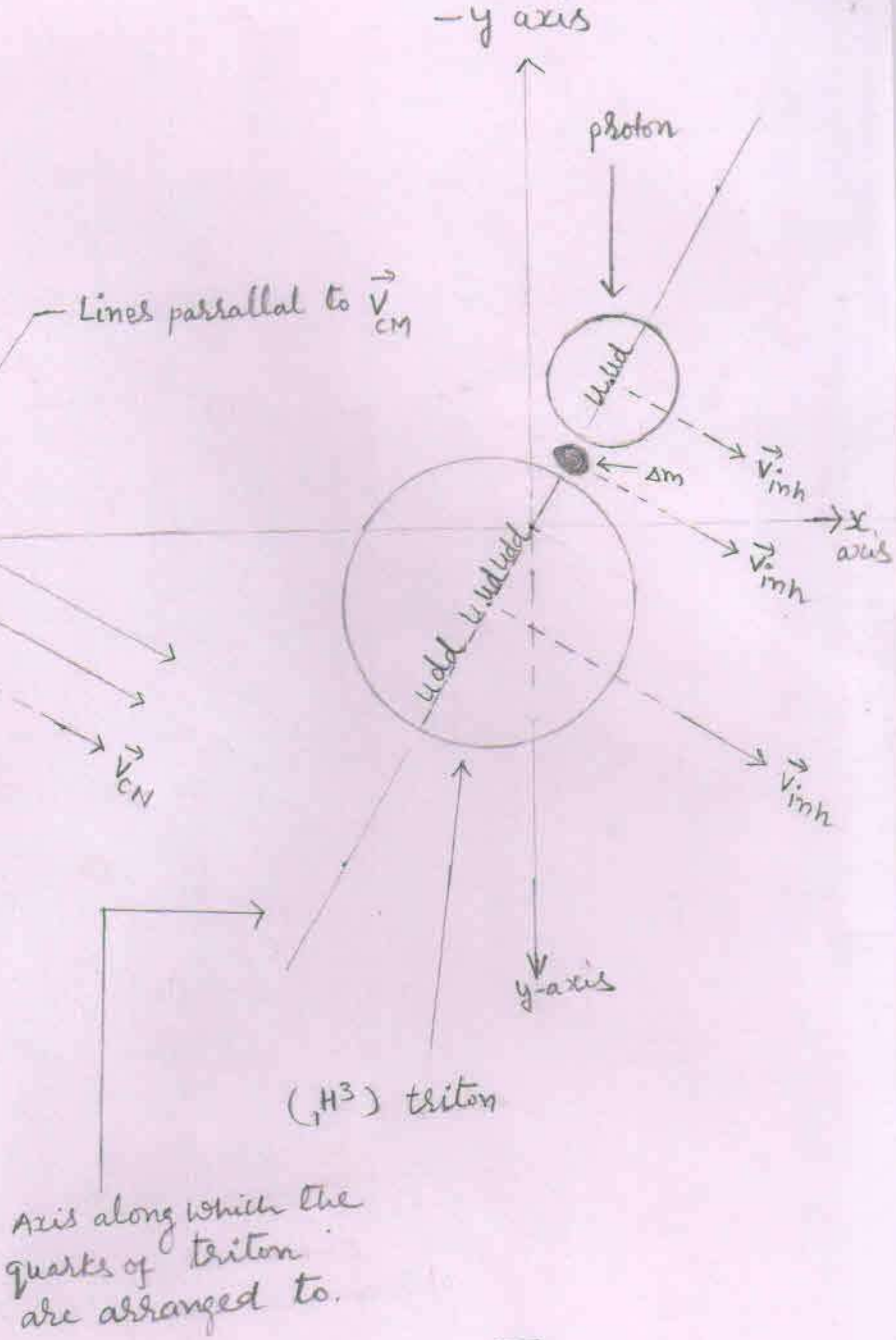
A heterogehous compound nucleus (showing lines parallal to $\vec{V}_{CM}$)



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All the three particles splits from the heterogenous compound nucleus with an inhereted velocity ($\vec{V}_{inn}$)

Components of inhereted velocity of the particle

Each particle has an inhereted velocity ($\vec{V}_{inh}$) equal to the velocity of Compound nucleus ($\vec{V}_{CN}$)

I. For triton ($_1H^3$)

$$\vec{V}_{inh} = \vec{V}_{CN} = 0.3089 \times 10^7 \ m/s$$

Components of inhereted velocity of the triton

1. $\vec{V}_x = V_{inh} \cos\alpha = V_{CN} \cos\alpha = 0.2676 \times 10^7 \ m/s$

2. $\vec{V}_y = V_{inh} \cos\beta = V_{CN} \cos\beta = 0.1545 \times 10^7 \ m/s$

3. $\vec{V}_z = V_{inh} \cos\gamma = V_{CN} \cos\gamma = 0 \quad m/s$

II. Inhereted velocity of the proton

$$\vec{V}_{inh} = \vec{V}_{CN} = 0.3089 \times 10^7 \ m/s$$

Components of inhereted velocity of the proton

1. $\vec{V}_x = V_{inh} \cos\alpha = V_{CN} \cos\alpha = 0.2676 \times 10^7 \ m/s$

2. $\vec{V}_y = V_{inh} \cos\beta = V_{CN} \cos\beta = 0.1545 \times 10^7 \ m/s$

3. $\vec{V}_z = V_{inh} \cos\gamma = V_{CN} \cos\gamma = 0 \quad m/s$

III. Inhereted velocity of the reduced mass

$$\vec{V} = V = 0.3089 \times 10^7 \ m/s$$

Propellation of particles

Reduced mass converts into energy and the total energy (E_T) propell both the particles with equal and opposite momentum.

1. Reduced mass

$$\Delta m = [m_d + m_d] - [m_t + m_p]$$

$$= [2 \times 2.01355] - [3.0155 + 1.00727] \text{ amu}$$

$$= [4.0271] - [4.02277] \text{ amu}$$

$$= 0.00433 \text{ amu}$$

$$= 0.00433 \times 1.6605 \times 10^{-27} \text{ Kg}$$

$$= 0.007189 \times 10^{-27} \text{ Kg}$$

2. Inhereted kinetic energy of reduced mass

$$E_{inh} = \frac{1}{2} \Delta m V_{CN}^2$$

$$= \frac{1}{2} \times 0.007189 \times 10^{-27} \times 0.0954800 1 \times 10^{14} \text{ J}$$

$$= 0.00034320289 \times 10^{-13} \text{ J}$$

$$= 0.000214 \text{ Mev}$$

3. Released energy (E_R) :

$$E_R = \Delta mc^2$$

$$= 0.00433 \times 931 \text{ Mev}$$

$$= 4.03123 \text{ Mev}$$

4. Total energy (E_T) :

$$E_T = E_{inh} + E_R$$

$$= 0.000214 + 4.03123 \text{ Mev}$$

$$= 4.031444 \text{ Mev}$$

Increased energy of the particle

The total energy E_T is divided between the particles according to their inverse masses. So, the increased energy (E_{inc}) of the particle

1. Increased energy of the triton

$$E_{inc} = \frac{m_p}{m_p + m_t} \times E_T$$

$$= \frac{1.00727 \text{ amu}}{[1.00727 + 3.0155] \text{ amu}} \times 4.031444 \text{ Mev}$$

$$= \frac{1.00727}{4.02277} \times 4.031444 \text{ Mev}$$

$$= [0.25039214272] \times 4.031444 \text{ Mev}$$

$$= 1.0094 \text{ Mev}$$

2. Increased energy of the proton

$$E_{inc} = [E_T] - [\text{Increased energy of triton}]$$

$$= [4.031444] - [1.0094] \text{ Mev}$$

$$= 3.022044 \text{ Mev}$$

Increased velocity of the particle

1. For triton

$$E_{inc} = \frac{1}{2} m_t v_{inc}^2$$

$$v_{inc} = \left[\frac{2 E_{inc}}{m_t} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 \times 1.0094 \times 1.6 \times 10^{-13} \text{ J}}{5.0072 \times 10^{-27} \text{ kg}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{3.23008 \times 10^{-13}}{5.0072 \times 10^{-27}} \right]^{\frac{1}{2}} \text{ m/s}$$

$$= [0.64508707461 \times 10^{14}]^{\frac{1}{2}} \text{ m/s}$$

$$= 0.8031 \times 10^{7} \text{ m/s}$$

2. For proton

$$v_{inc} = \left[\frac{2 E_{inc}}{m_p} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 \times 3.022044 \times 1.6 \times 10^{-13} \text{ J}}{1.6726 \times 10^{-27} \text{ kg}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{9.6705408 \times 10^{-13}}{1.6726 \times 10^{-27}} \right]^{\frac{1}{2}} \text{ m/s}$$

$$= [5.7817414 8033 \times 10^{14}]^{\frac{1}{2}} \text{ m/s}$$

$$= 2.4045 \times 10^{7} \text{ m/s}$$

Angle of propellation

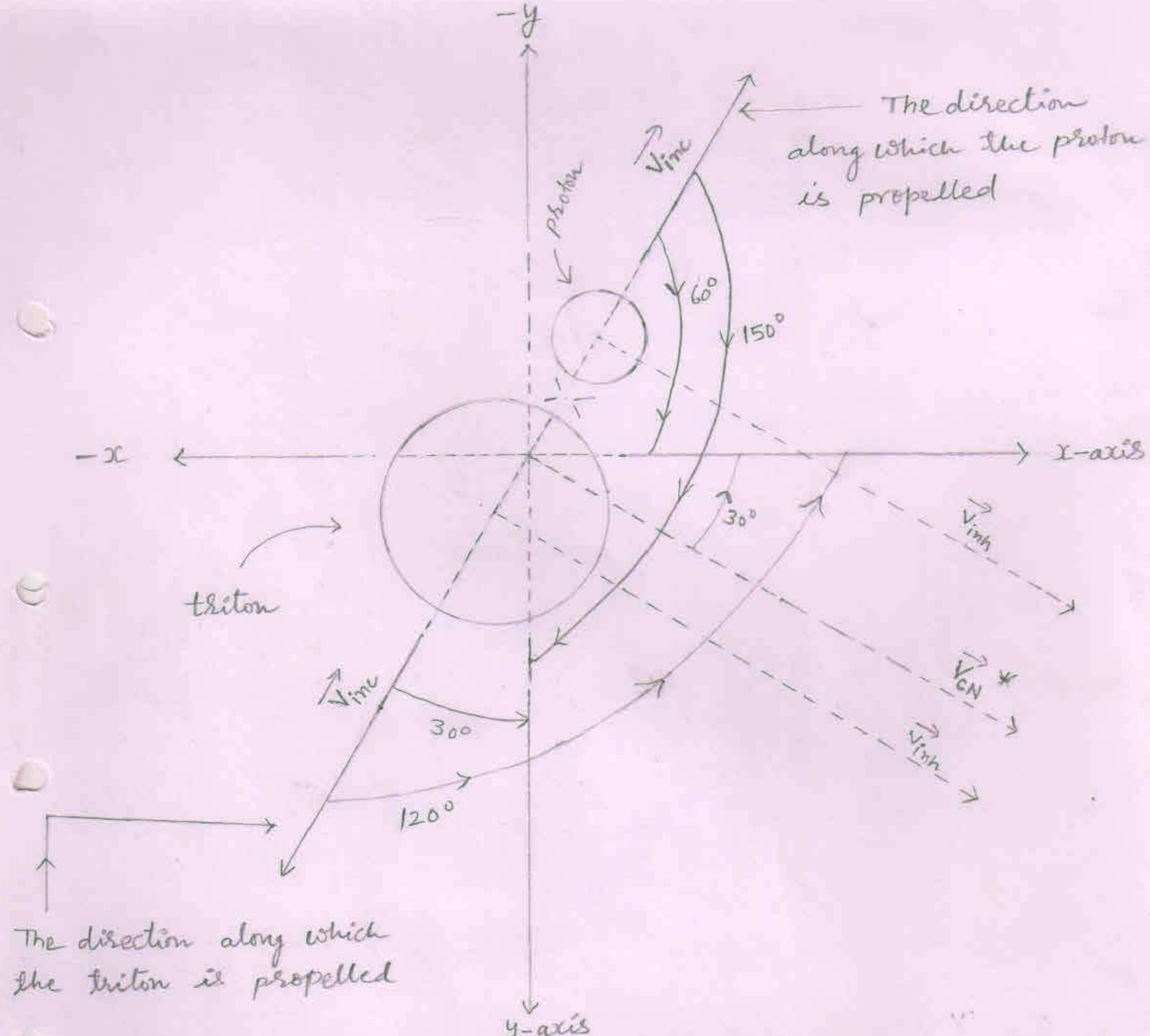
1. As the reduced mass converts into energy, the total energy (E_T) propell both the particles with equal and opposite momentum according to a ray (line) perpendicular to the direction (line) of the velocity of the compound nucleus ($\vec{V}_{CN}$).

2. We know that when there a fusion process occures, then we find the lighter nucleus in the forward direction [or in the direction of ion beam or in the direction of the velocity of the compound nucleus ($\vec{V}_{CN}$).]

3. At point 'F', as $\vec{V}_{CN}$ makes 30° angle with x-axis, 60° angle with y-axis and 90° angle with z-axis.

4. So, the proton is propelled making 60° angle with x-axis, 150° angle with y-axis and 90° angle with z-axis.

5. While the triton is propelled making 120° angle with x-axis, 30° angle with y-axis and 90° angle with z-axis.



and $\vec{V}_{inc} \perp \vec{V}_{CN}^{*}$

$\Rightarrow$ $\vec{V}_{inh} = \vec{V}_{CN}$

$\Rightarrow$ The direction along which the proton is propelled make angle 180° With the direction along which the triton is propelled.

$\Rightarrow$ As a heterogehous compound nucleus splits into three particles according to the lines parallal to $\vec{V}_{CN}$. So, the produced both finite nuclei are propelled according to a ray line perpendicular to ... alter splitting of a compound nucleus, Components of increased velocity of the particle

I For proton

1. $\vec{v}_x = v_{inc} \cos\alpha$

$v_{inc} = 2.4045 \times 10^7$ m/s

$\cos\alpha = \cos 60^\circ = \frac{1}{2}$

$\vec{v}_x = 2.4045 \times 10^7 \times \frac{1}{2}$ m/s

$= 1.2022 \times 10^7$ m/s

2. $\vec{v}_y = v_{inc} \cos\beta$

$\cos\beta = \cos 150^\circ = -\frac{\sqrt{3}}{2}$

$\vec{v}_y = 2.4045 \times 10^7 \times \left(-\frac{\sqrt{3}}{2}\right)$ m/s

$= 2.4045 \times 10^7 \times (-0.866)$ m/s

$= -2.0822 \times 10^7$ m/s

3. $\vec{v}_z = v_{inc} \cos\gamma = v_{inc} \cos 90^\circ = v_{inc} \times 0 = 0$ m/s

II. For triton

1. $\vec{v}_x = v_{inc} \cos\alpha$

$v_{inc} = 0.8031 \times 10^7$ m/s

$\cos\alpha = \cos 120^\circ = -\frac{1}{2}$

$\vec{v}_x = 0.8031 \times 10^7 \times \left(-\frac{1}{2}\right)$ m/s

$= -0.4015 \times 10^7$ m/s

2. $\vec{v}_y = v_{inc} \cos\beta$

$\cos\beta = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\vec{v}_y = 0.8031 \times 10^7 \times \frac{\sqrt{3}}{2}$ m/s

$= 0.8031 \times 10^7 \times (0.866)$ m/s

$= 0.6954 \times 10^7$ m/s

3. $\vec{v}_z = v_{inc} \cos\gamma = v_{inc} \cos 90^\circ = v_{inc} \times 0 = 0$ m/s

Components of final velocity of the particle

1. For triton

According to	Inhereted velocity	Increased velocity	Final Velocity (Inh + Inc)
X-axis	$\vec{v}_x = 0.2676 \times 10^7$ m/s	$\vec{v}_x = -0.4015 \times 10^7$ m/s	$\vec{v}_x = -0.1339 \times 10^7$ m/s
Y-axis	$\vec{v}_y = 0.1545 \times 10^7$ m/s	$\vec{v}_y = 0.6954 \times 10^7$ m/s	$\vec{v}_y = 0.8499 \times 10^7$ m/s
Z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

2. For proton

According to	Inhereted velocity	Increased velocity	Final Velocity (Inh + Inc)
X-axis	$\vec{v}_x = 0.2676 \times 10^7$ m/s	$\vec{v}_x = 1.2022 \times 10^7$ m/s	$\vec{v}_x = 1.4698 \times 10^7$ m/s
Y-axis	$\vec{v}_y = 0.1545 \times 10^7$ m/s	$\vec{v}_y = -2.0822 \times 10^7$ m/s	$\vec{v}_y = -1.9277 \times 10^7$ m/s
Z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

components of final momentum of the triton

1. $\vec{P}_x = m_t \vec{v}_x$

$= 5.0072 \times 10^{-27} \times (-0.1399 \times 10^{7})$ Kg m/s

$= -0.7005 \times 10^{-20}$ Kg m/s

2. $\vec{P}_y = m_t \vec{v}_y$

$= 5.0072 \times 10^{-27} \times 0.8499 \times 10^{7}$ Kg m/s

$= 4.2556 \times 10^{-20}$ Kg m/s

3. $\vec{P}_z = m_t \vec{v}_z$

$= m_t \times 0$ Kg m/s

$= 0$ Kg m/s

Final kinetic energy of the particle – triton

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$= (0.1339 \times 10^7)^2 + (0.8499 \times 10^7)^2 + (0)^2 \ m^2/s^2$$

$$= (0.01792921 \times 10^{14}) + (0.72233001 \times 10^{14}) + 0 \ m^2/s^2$$

$$\Rightarrow v^2 = 0.74025922 \times 10^{14} \ m^2/s^2$$

$$\Rightarrow v = 0.8603 \times 10^7 \ m/s$$

$$\Rightarrow mv^2 = 5.0072 \times 10^{-27} \times 0.74025922 \times 10^{14} \ J$$
$$= 3.70662596638 \times 10^{-13} \ J$$

$$\Rightarrow K.E = \frac{1}{2} mv^2 = 1.85331298319 \times 10^{-13} \ J$$
$$= 1.1583 \ MeV$$

⇒ Angles made by the triton with respect to axes x, y, z when it is at point F. :–
Triton is produced at point F. If α, β, γ are the angles made by the triton with respect to axes x, y, z respectively. Then

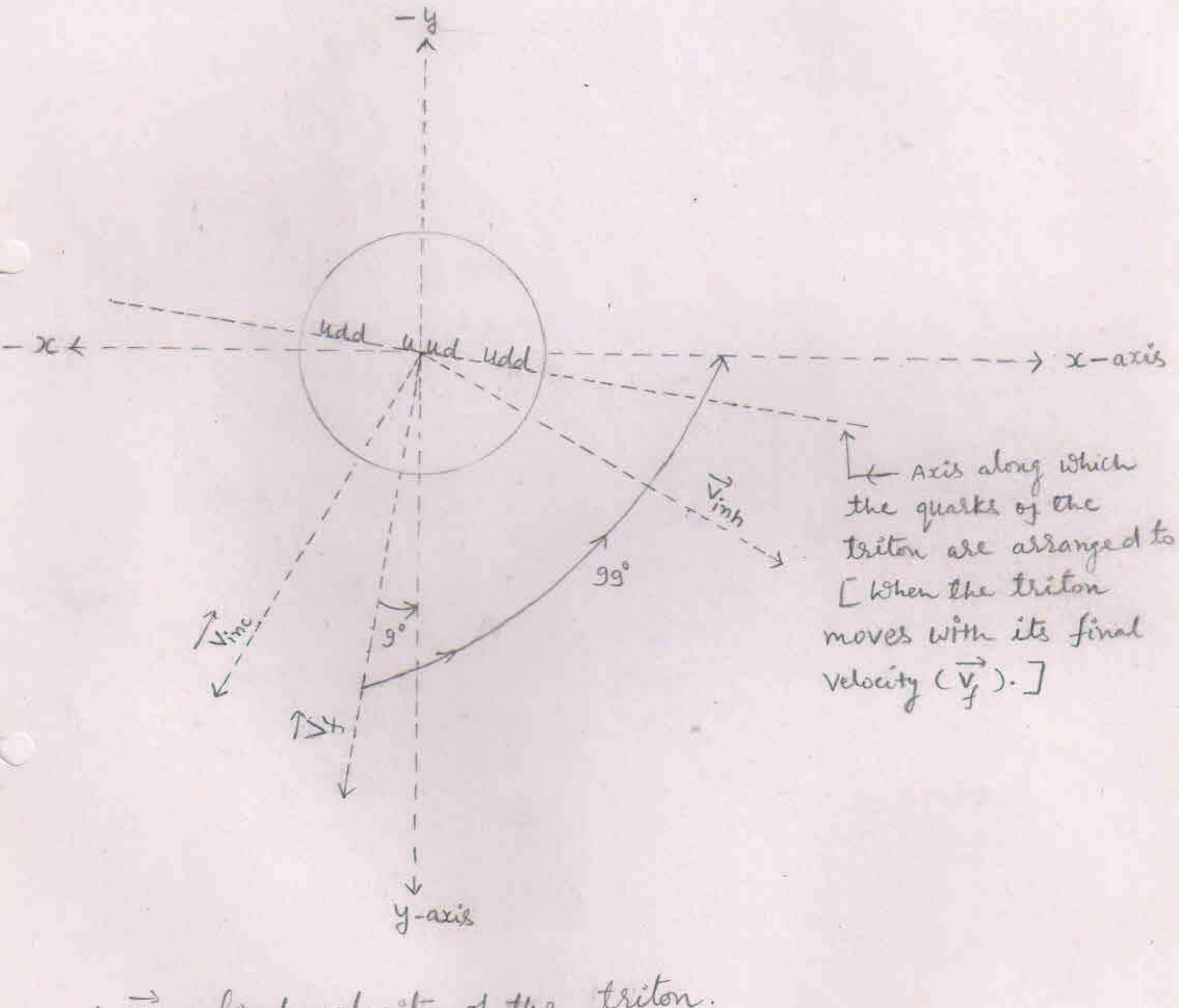
1. $\cos\alpha = \dfrac{v_x}{v} = \dfrac{v\cos\alpha}{v} = \dfrac{-0.1339 \times 10^7}{0.8603 \times 10^7} = -0.1556$

$\alpha \simeq 99^\circ$

2. $\cos\beta = \dfrac{v_y}{v} = \dfrac{v\cos\beta}{v} = \dfrac{0.8499 \times 10^7}{0.8603 \times 10^7} = 0.9879$

$\beta \simeq 9^\circ$

3. $\cos\gamma = \dfrac{v_z}{v} = \dfrac{v\cos\gamma}{v} = \dfrac{0}{v} = 0$



$\Rightarrow$ $\vec{v}_f$ = final velocity of the triton.

$$\Rightarrow \quad \vec{V}_f = \vec{V}_{\circ\, inh} + \vec{V}_{\circ\, inc}$$

Acting forces on the triton

1. $F_y = q V_x B_z \sin\theta$

$V_x = 0.1339 \times 10^7$ m/s
$B_z = 1$ Tesla
$\sin\theta = \sin 90^\circ = 1$

$F_y = 1.6 \times 10^{-19} \times 0.1339 \times 10^7 \times 1 \times 1$ N
$= 0.2142 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_y is according to $+y$ axis.
So,
$F_y = 0.2142 \times 10^{-12}$ N

2. $F_x = q V_y B_z \sin\theta$

$V_y = 0.8499 \times 10^7$ m/s
$\sin\theta = \sin 90^\circ = 1$

$F_x = 1.6 \times 10^{-19} \times 0.8499 \times 10^7 \times 1 \times 1$ N
$= 1.3598 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_x is according to $+x$ axis.
So,
$\vec{F_x} = 1.3598 \times 10^{-12}$ N

3. $F_z = q V_x B_y \sin\theta$

$B_y = 1$ Tesla
$\sin\theta = \sin 90^\circ = 1$

$F_z = 1.6 \times 10^{-19} \times 0.1339 \times 10^7 \times 1 \times 1$ N
$= 0.2142 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_z is according to $+z$ axis.
So,
$\vec{F_z} = 0.2142 \times 10^{-12}$ N

Resultant force (F_R)

$F_R^2 = F_x^2 + F_y^2 + F_z^2$

$F_x = 1.3598 \times 10^{-12}$ N

$F = F_y = F_z = 0.2142 \times 10^{-12}$ N

$F_R^2 = F_x^2 + 2F^2$

$= (1.3598 \times 10^{-12})^2 + 2(0.2142 \times 10^{-12})^2$ N^2

$= (1.84905604 \times 10^{-24}) + 2(0.04588164 \times 10^{-24}) N^2$

$F_R^2 = 1.94081932 \times 10^{-24}$ N^2

$F_R = 1.3931 \times 10^{-12}$ N

Radius of the circular path :

Resultant force acts as a centripetal force on the triton. So, the triton follows a confined circular path.

The radius of the circular orbit obtained by the triton is —

$r = \dfrac{mv^2}{F_R}$

$= \dfrac{3.7066 \times 10^{-13}}{1.3931 \times 10^{-12}} \dfrac{J}{N}$

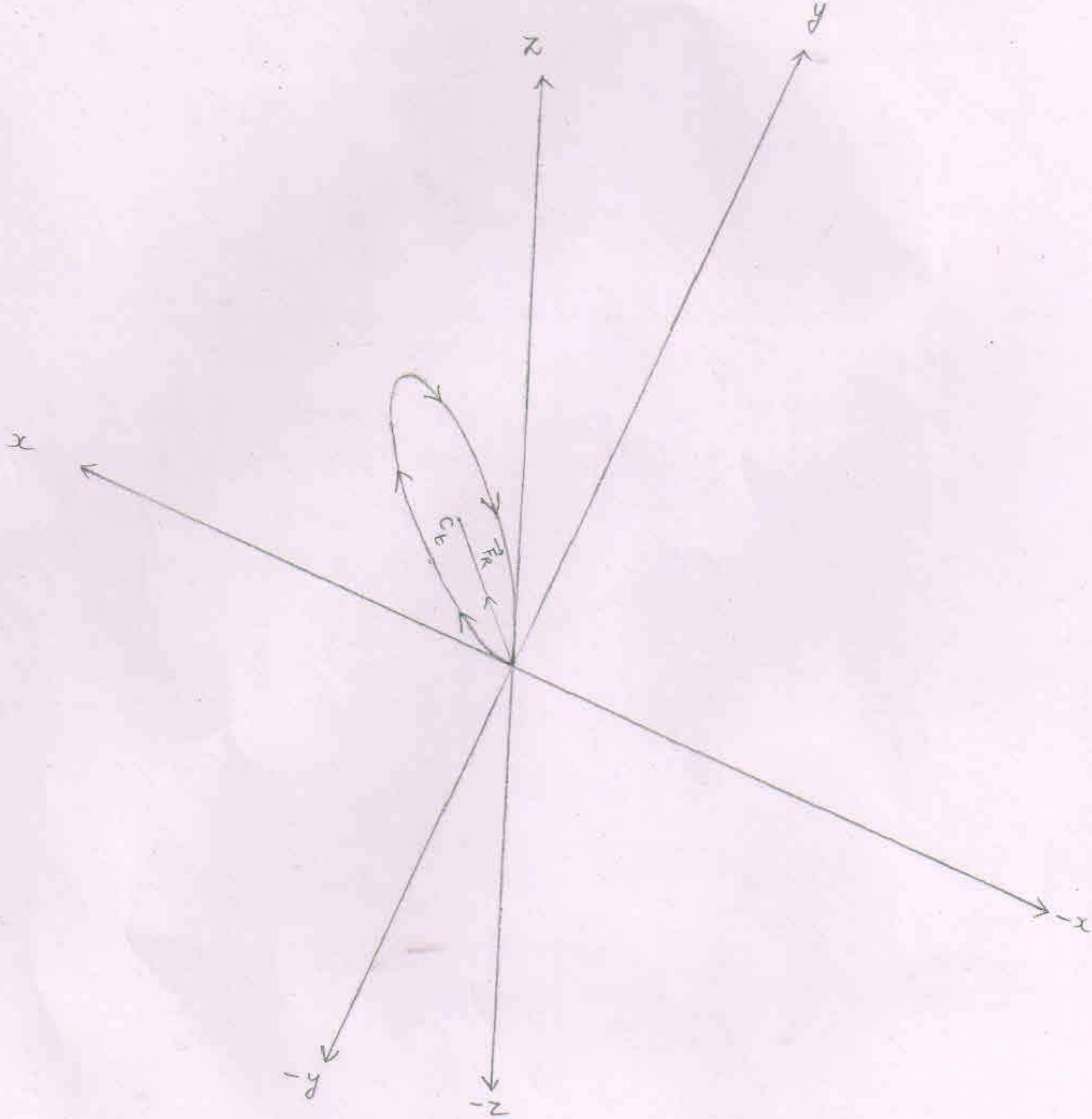
$= 2.6606 \times 10^{-1}$ m

$= 26.60$ cm

⇒ The circular orbit followed by the confined triton lies in the I (up) quadrant made up of positive x axis, positive y axis and the positive z axis.

⇒ $\vec{F_R}$ = The resultant force acting on the particle (at point F) towards the centre of the circle.

⇒ c_t = centre of the circle obtained by the triton



Angles that make the resultant force ($\vec{F_R}$) [acting on the triton when the triton is at point 'F'] with positive x, y and z-axes.

1. With x-axis

$$\cos\alpha = \frac{F_R \cos\alpha}{F_R} = \frac{\vec{F_x}}{F_R} = \frac{1.3598 \times 10^{-12}}{1.3931 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\alpha = 0.9760$

$\Rightarrow \alpha \simeq 12.6$ degree [$\because \cos(12.6) = 0.9759$]

2. With y-axis

$$\cos\beta = \frac{F_R \cos\beta}{F_R} = \frac{\vec{F_y}}{F_R} = \frac{0.2142 \times 10^{-12}}{1.3931 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\beta = 0.1537$

$\Rightarrow \beta \simeq 81.2$ degree [$\because \cos(81.2) = 0.1529$]

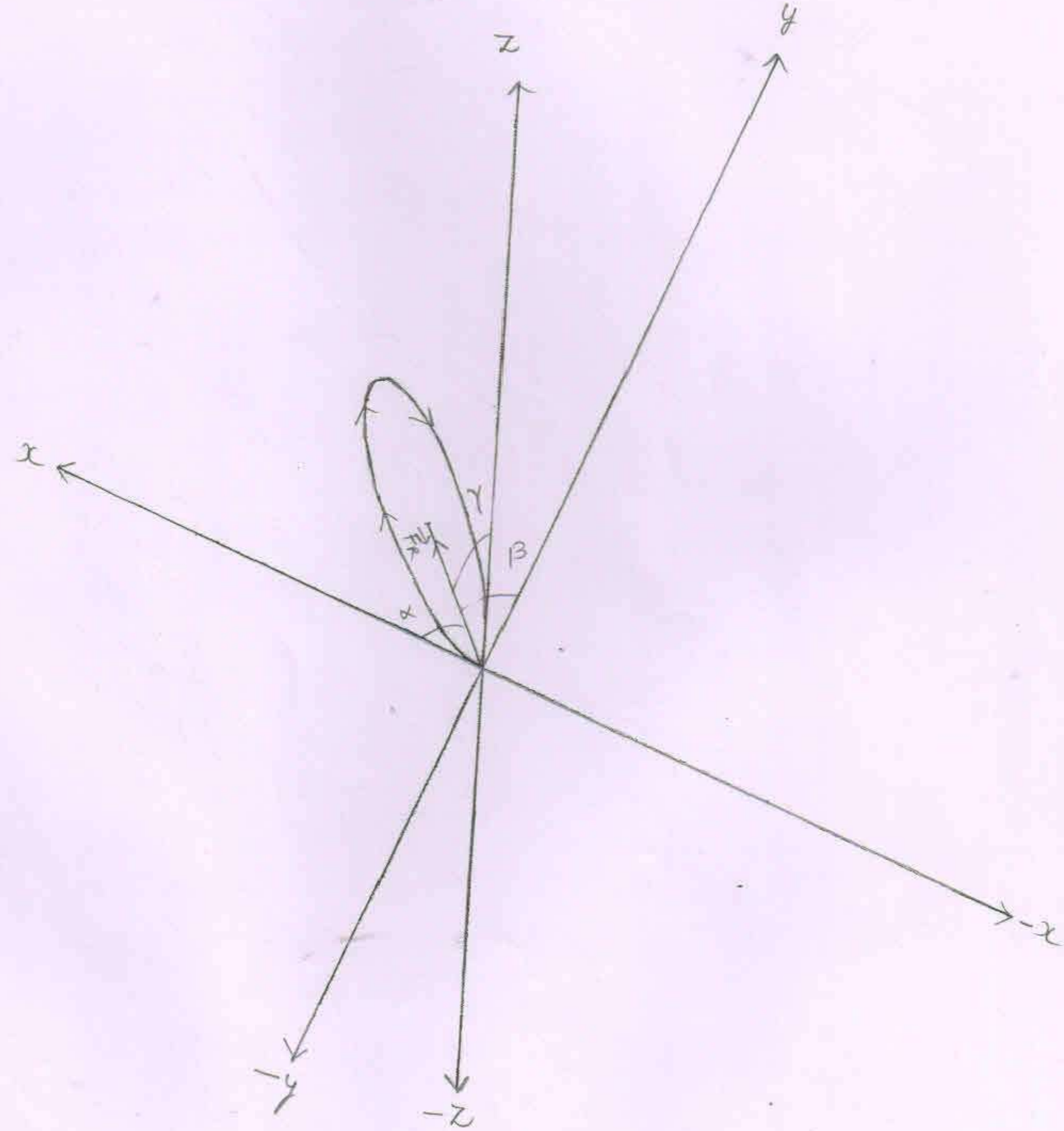
3. With z-axis

$$\cos\gamma = \frac{F_R \cos\gamma}{F_R} = \frac{\vec{F_z}}{F_R} = \frac{0.2142 \times 10^{-12}}{1.3931 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\gamma = 0.1537$

$\Rightarrow \gamma \simeq 81.2$ degree

Angles that make the resultant force ($\overrightarrow{F_R}$) at point F with positive x, y and z axes.



Where,

$\alpha \simeq 12.6$

$\beta \simeq 81.2$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the triton

1. $\cos\alpha = \dfrac{x_2 - x_1}{d}$

$d = 2 \times r$
$= 2 \times 26.606 \times 10^{-2}\ m$
$= 53.212 \times 10^{-2}\ m$

$\cos\alpha = 0.97$

$\Rightarrow x_2 - x_1 = d \times \cos\alpha$
$\Rightarrow x_2 - x_1 = 53.212 \times 10^{-2} \times 0.97\ m$
$\Rightarrow x_2 - x_1 = 51.6156 \times 10^{-2}\ m$
$\Rightarrow x_2 = 51.6156 \times 10^{-2}\ m$

2. $\cos\beta = \dfrac{y_2 - y_1}{d}$

$\cos\beta = 0.15$

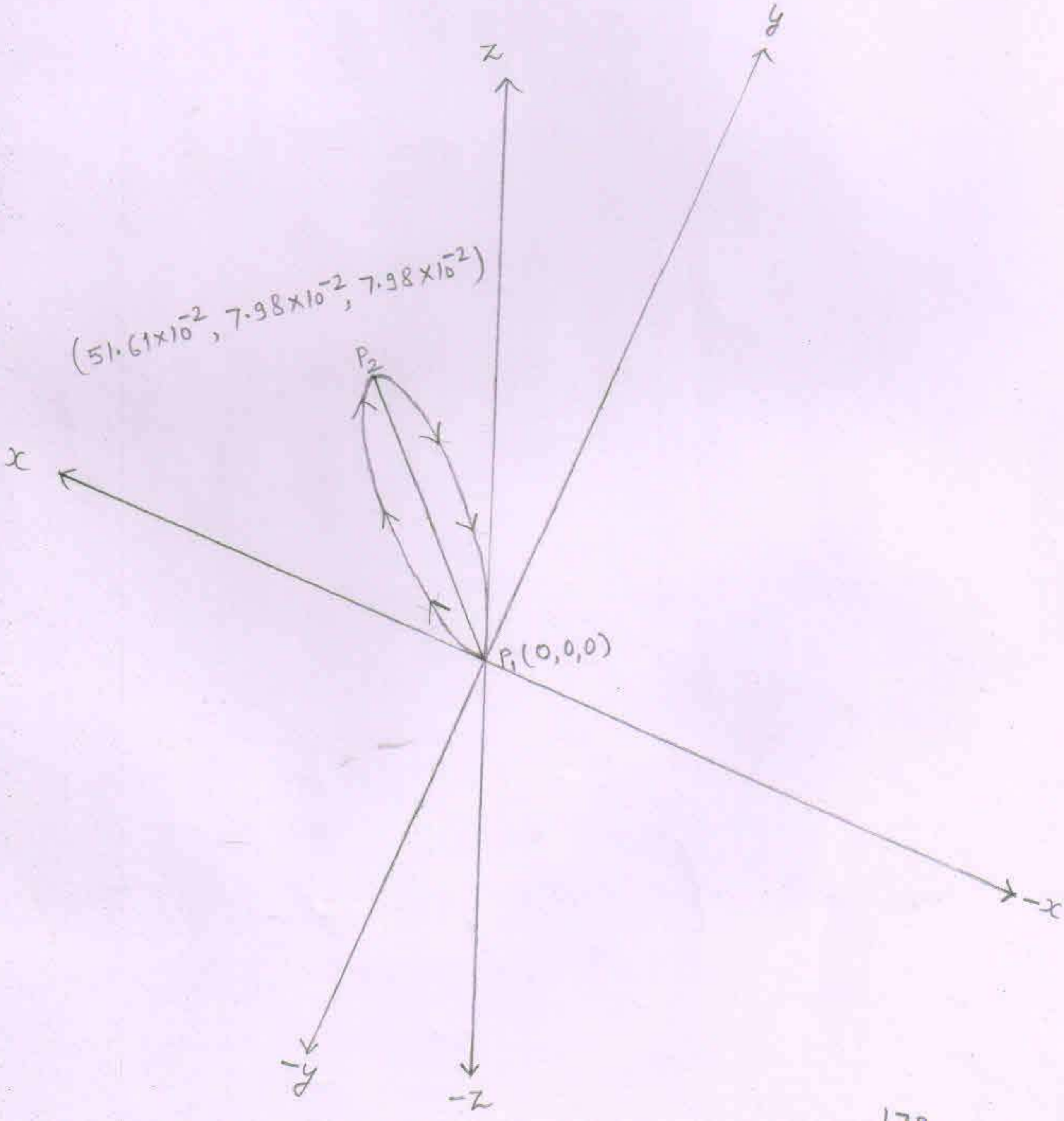
$\Rightarrow y_2 - y_1 = d \times \cos\beta$
$\Rightarrow y_2 - y_1 = 53.212 \times 10^{-2} \times 0.15\ m$
$\Rightarrow y_2 - y_1 = 7.9818 \times 10^{-2}\ m$
$\Rightarrow y_2 = 7.9818 \times 10^{-2}\ m$ $[\because y_1 = 0]$

3. $\cos\gamma = \dfrac{z_2 - z_1}{d}$

$\cos\gamma = 0.15$

$\Rightarrow z_2 - z_1 = d \times \cos\gamma$
$\Rightarrow z_2 - z_1 = 53.212 \times 10^{-2} \times 0.15\ m$
$\Rightarrow z_2 - z_1 = 7.9818 \times 10^{-2}\ m$
$\Rightarrow z_2 = 7.9818 \times 10^{-2}\ m$ $[\because z_1 = 0]$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2 z_2)$



Time period of the particle - the triton

$$T = \frac{2\pi r}{v}$$

$r = 26.60 \times 10^{-2}$ m
$v = 0.8603 \times 10^{7}$ m/s

$$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 26.60 \times 10^{-2}}{0.8603 \times 10^{7}} \text{ s}$$

$$= \frac{167.048 \times 10^{-2}}{0.8603 \times 10^{7}} \text{ s}$$

$$= 194.174 \times 10^{-9} \text{ s}$$

$$= 1.94 \times 10^{-7} \text{ second}$$

Time of Confinement of triton

$$\Rightarrow t_e = \frac{3c^5 m^3}{4e^4 B^2}$$

$c = 3\times10^{10}$ cm/s

$m = 5.0072 \times 10^{-24}$ gram

$e = 4.8\times10^{-10}$

$B = 10^4$ Gauss

$$\Rightarrow t_e = \frac{3\times(3\times10^{10})^5 \times (5.0072\times10^{-24})^3}{4\times(4.8\times10^{-10})^4 \times (10^4)^2} \text{ seconds}$$

$$= \frac{3\times 243\times10^{50} \times 125.5407\times10^{-72}}{4\times 530.84\times10^{-40}\times10^8} \text{ seconds}$$

$$= \frac{91519.2271\times10^{-22}}{2123.36\times10^{-32}} \text{ seconds}$$

$$= 43.10\times10^{10} \text{ seconds}$$

$$\simeq 4.31\times10^{11} \text{ seconds}$$

Final kinetic energy of the particle - proton

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

$$= (1.4698 \times 10^7)^2 + (1.9277 \times 10^7)^2 + (0)^2 \ m^2/s^2$$

$$= (2.16031204 \times 10^{14}) + (3.71602729 \times 10^{14}) + 0 \ m^2/s^2$$

$$\Rightarrow V^2 = 5.87633933 \times 10^{14} \ m^2/s^2$$

$$\Rightarrow mv^2 = 1.6726 \times 10^{-27} \times 5.87633933 \times 10^{14} \ J$$
$$= 9.82876516335 \times 10^{-13} \ J$$

$$\Rightarrow K.E = \frac{1}{2} mv^2 = 4.914382258167 \times 10^{-13} \ J$$
$$= 3.0714 \ \text{Mev}$$

Acting forces on the proton

1. $F_y = q v_x B_z \sin\theta$

$v_x = 1.4698 \times 10^7$

$B_z = 1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$F_y = 1.6 \times 10^{-19} \times 1.4698 \times 10^7 \times 1 \times 1$ N

$= 2.3516 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_y is according to $-y$ axis.

So,

$\vec{F}_y = -2.3516 \times 10^{-12}$ N

2. $F_z = q v_x B_y \sin\theta$

$B_y = 1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$F_z = 1.6 \times 10^{-19} \times 1.4698 \times 10^7 \times 1 \times 1$ N

$= 2.3516 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_z is according to $-z$ axis.

So,

$\vec{F}_z = -2.3516 \times 10^{-12}$ N

3. $F_x = q v_y B_z \sin\theta$

$v_y = 1.9277 \times 10^7$ m/s

$\sin\theta = \sin 90^\circ = 1$

$F_x = 1.6 \times 10^{-19} \times 1.9277 \times 10^7 \times 1 \times 1$ N

$= 3.0843 \times 10^{-12}$ N

From the right hand palm rule, the direction of force F_x is according to $-x$ axis.

So,

$\vec{F}_x = -3.0843 \times 10^{-12}$ N

Resultant force (F_R)

$F_R^2 = F_x^2 + F_y^2 + F_z^2$

$F_x = 3.0843 \times 10^{-12}$ N

$F = F_y = F_z = 2.3516 \times 10^{-12}$ N

$F_R^2 = F_x^2 + 2F^2$

$= (3.0843 \times 10^{-12})^2 + 2(2.3516 \times 10^{-12})^2 \quad N^2$

$= (9.51290649 \times 10^{-24}) + 2(5.53002256 \times 10^{-24}) N^2$

$F_R^2 = 20.57295161 \times 10^{-24} \quad N^2$

$F_R = 4.5357 \times 10^{-12} \quad N$

Radius of the circular path :

Resultant force acts as a centripetal force on the proton. So, the proton tries to follow a confined circular path.

Radius of the circular orbit obtained by the proton is —

$r = \dfrac{mv^2}{F_R}$

$= \dfrac{9.8287 \times 10^{-13}}{4.5357 \times 10^{-12}} \dfrac{J}{N}$

$= 2.1669 \times 10^{-1} \quad m$

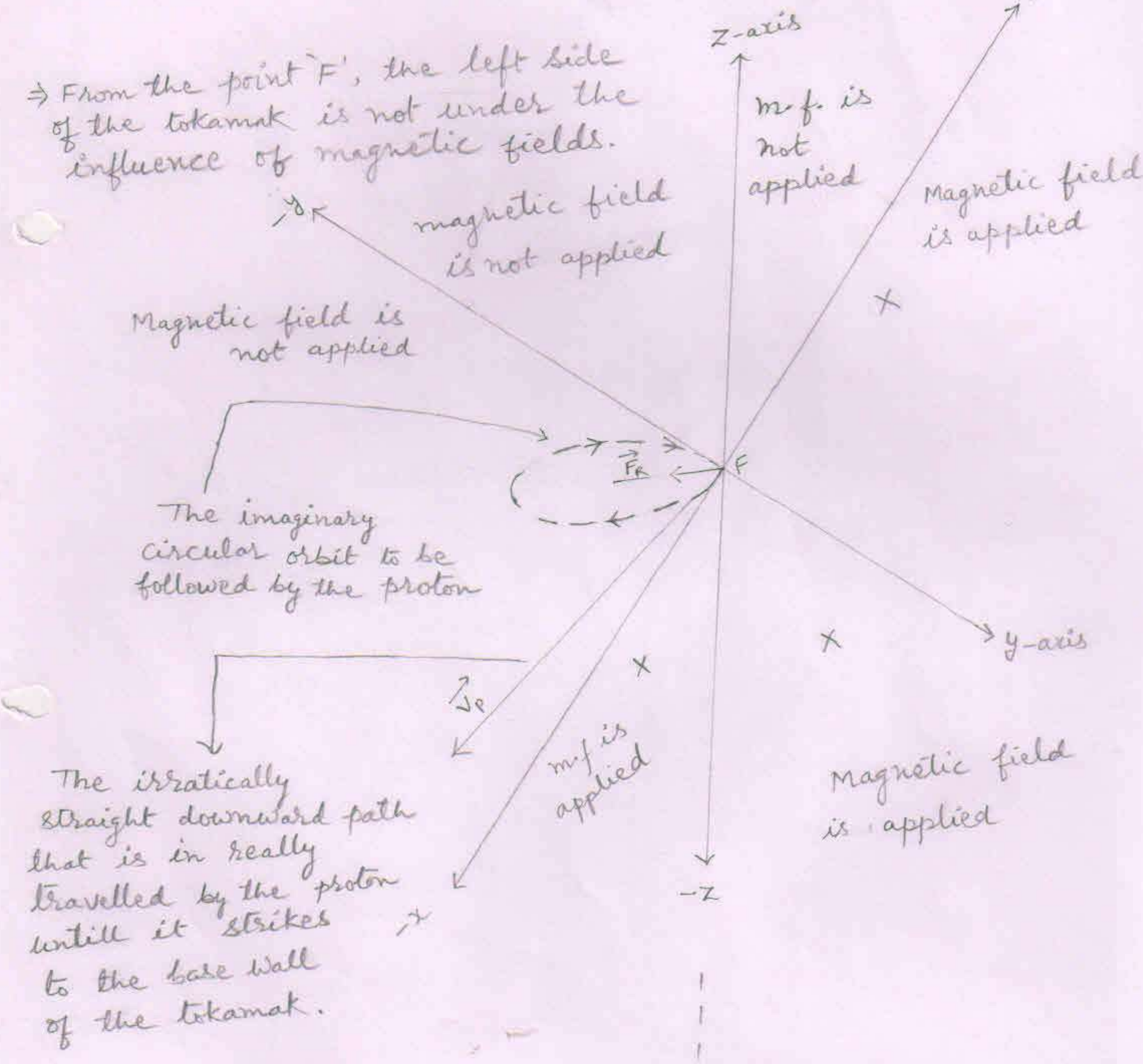
$= 21.66$ cm

Conclusion :-

The components of the resultant force [$\vec{F}_x$, $\vec{F}_y$ and $\vec{F}_z$] that are acting on the proton have negative directions.

The resultant force ($\vec{F}_R$) acting on the proton at point 'F', tends the proton to undergo the confined circular orbit of radius 21.66 cm but in trying to follow the confined circular path, the proton get rid of the region covered-up by the applied magnetic fields.

So, the proton starts its circular motion from point 'F' and as it travel a negligible circular path (distance), it get rid of the region covered-up by the applied magnetic fields. So, then it travels irratically straight downward and strike to the base wall of the tokamak.

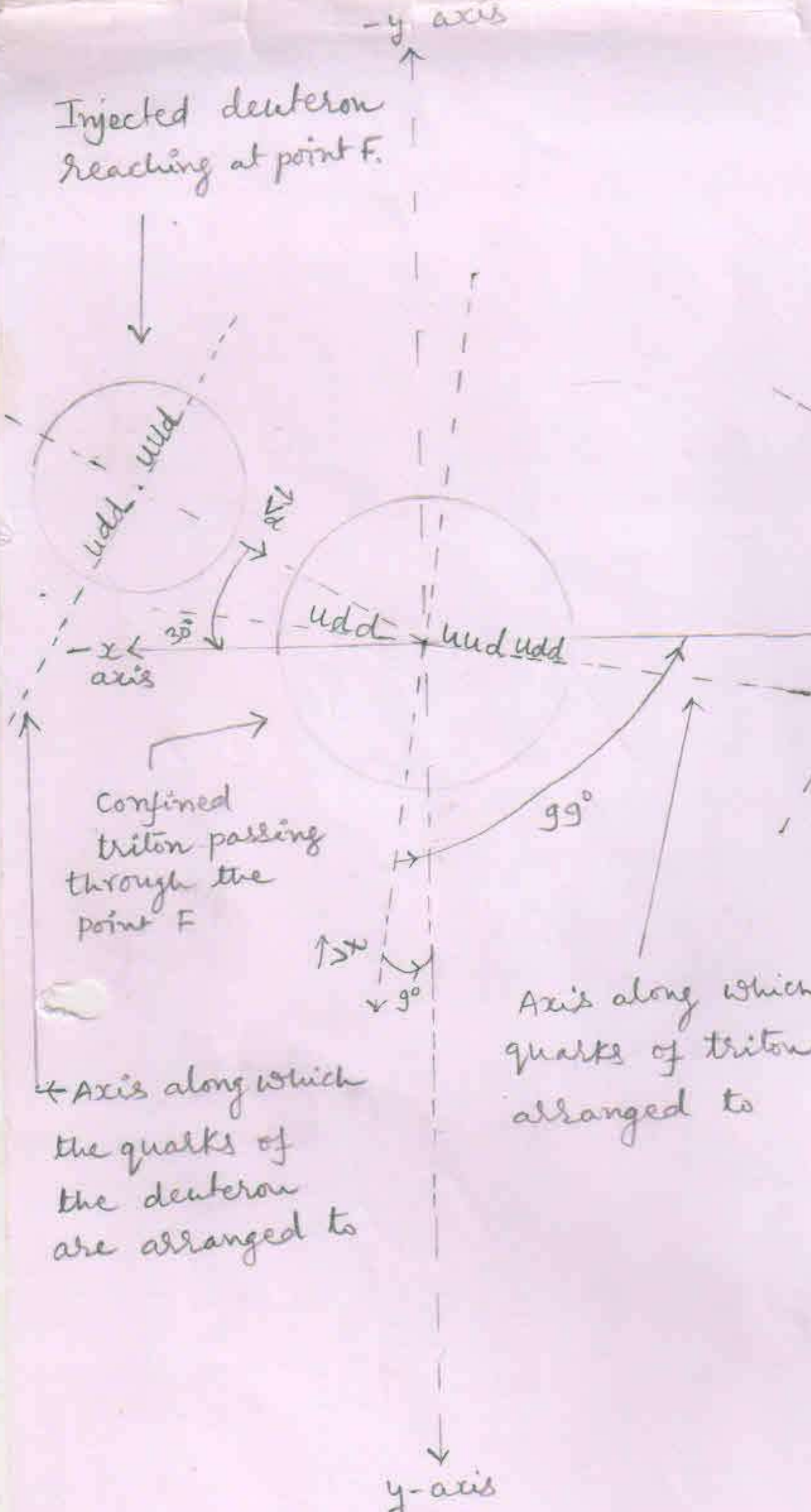


⇒ In trying to follow the confined circular orbit, the proton reaches in a region made-up of negative x-axis and negative y-axis where the magnetic fields are not applied. So, as the proton get rid of the magnetic fields, it starts its linear motion leaving the circular motion.

For fusion reaction : ${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n$

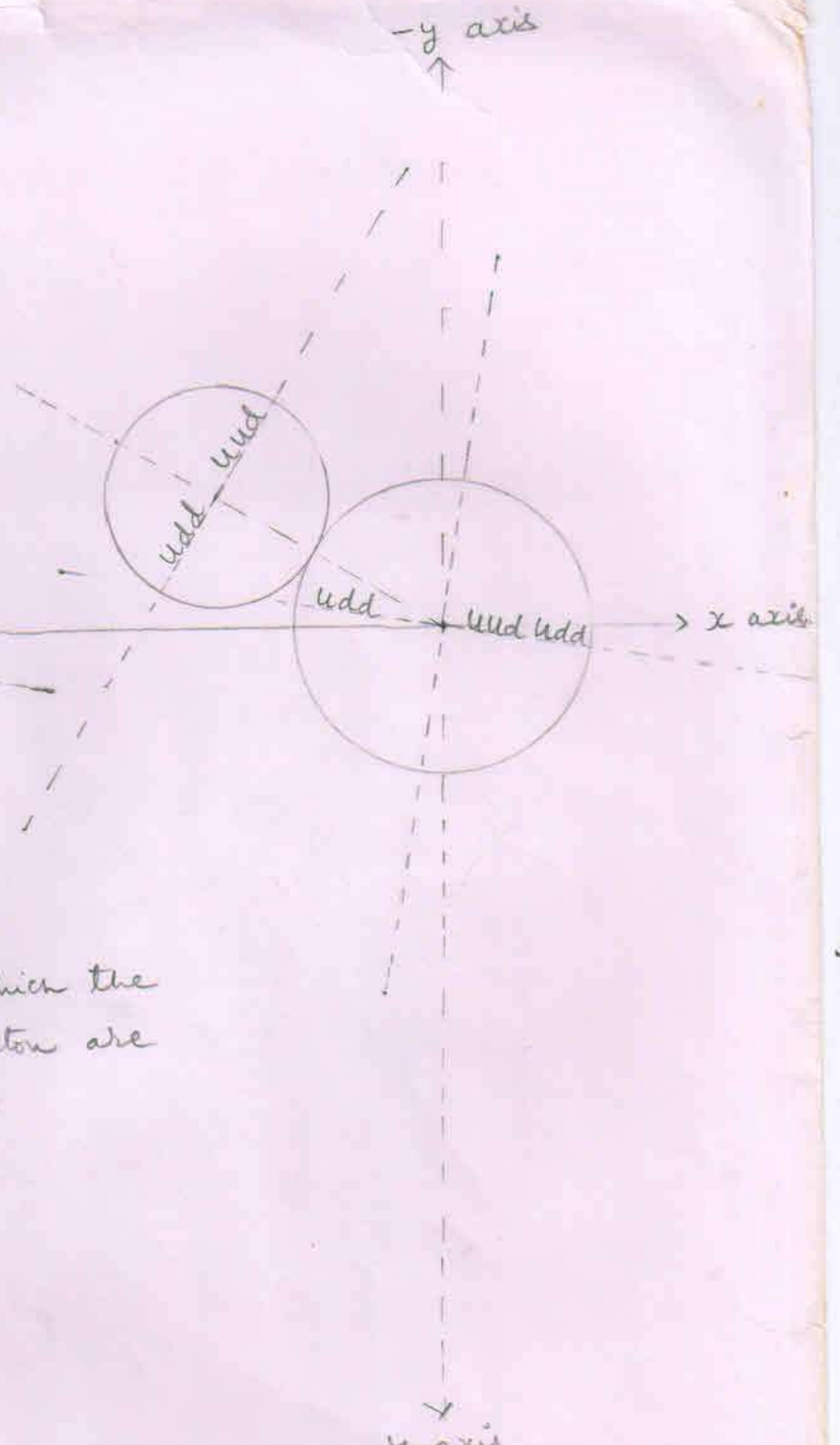
1. Interaction of nuclei :-

The injected deuteron reaches at point 'F' and interacts [experiences a repulsive force due to confined triton] with the confined triton passing through the point 'F'. The injected deuteron overcomes the electrostatic repulsive force and - a like two solid spheres join - the injected deuteron dissimilarly joins with the confined triton.



1

Injection of deutron



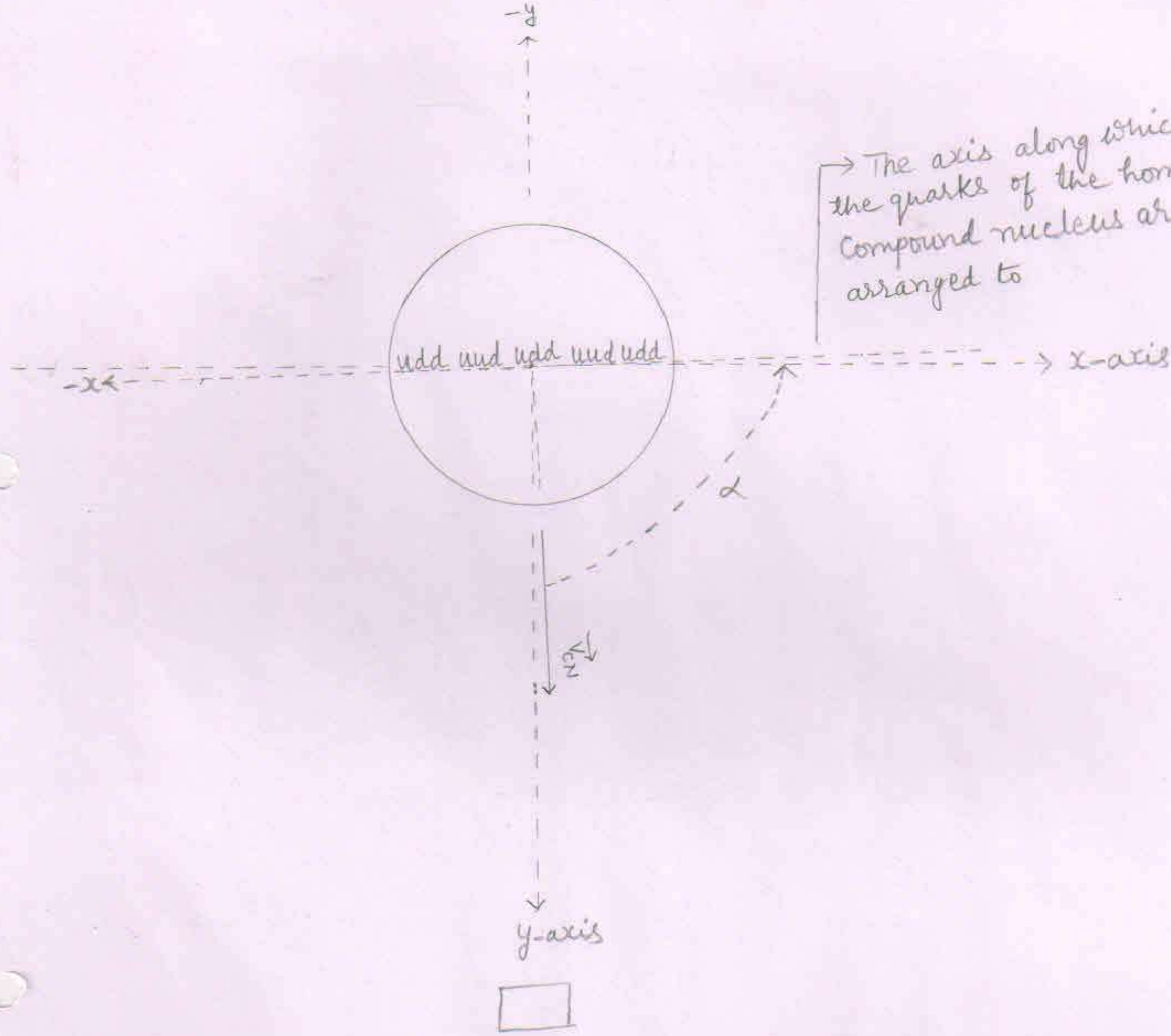
2

Interaction of nuclei

2. Formation of a homogenous compound nucleus:- The constituents (quarks and gluons) of the dissimilarly joined nuclei (deuteron and triton) behave like a liquid and form a homogenous compound nucleus having similarly distributed groups of quarks with similarly distributed surrounding gluons.

Thus within a homogenous compound nucleus - each group of quarks is surrounded by gluons in equal proportion. So, within a homogenous compound nucleus there are 5 groups of quarks surrounded by the gluons.

The homogenous Compound nucleus



$\Rightarrow$ where,

$\alpha \simeq 87.8$

$\beta \simeq 2.2$

$\gamma = 90^\circ$

$\Rightarrow$ $\vec{V}_{CN}$ = velocity of Compound nucleus

$\Rightarrow$ α, β, and γ are the angles that make the velocity of Compound nucleus with positive x, y and Z axes respectively.

3. Formation of lobes within into the homogenous compound nucleus or the transformation of the homogenous compound nucleus into the heterogenous compound nucleus :-

The central group of quarks with its surrounding gluons to become a stable and the next higher nucleus (the helion-4) than the reactant one (the triton) includes the other three (nearby located) groups of quarks with their surrounding gluons and rearrange to form the 'A' lobe of the heterogenous compound nucleus.

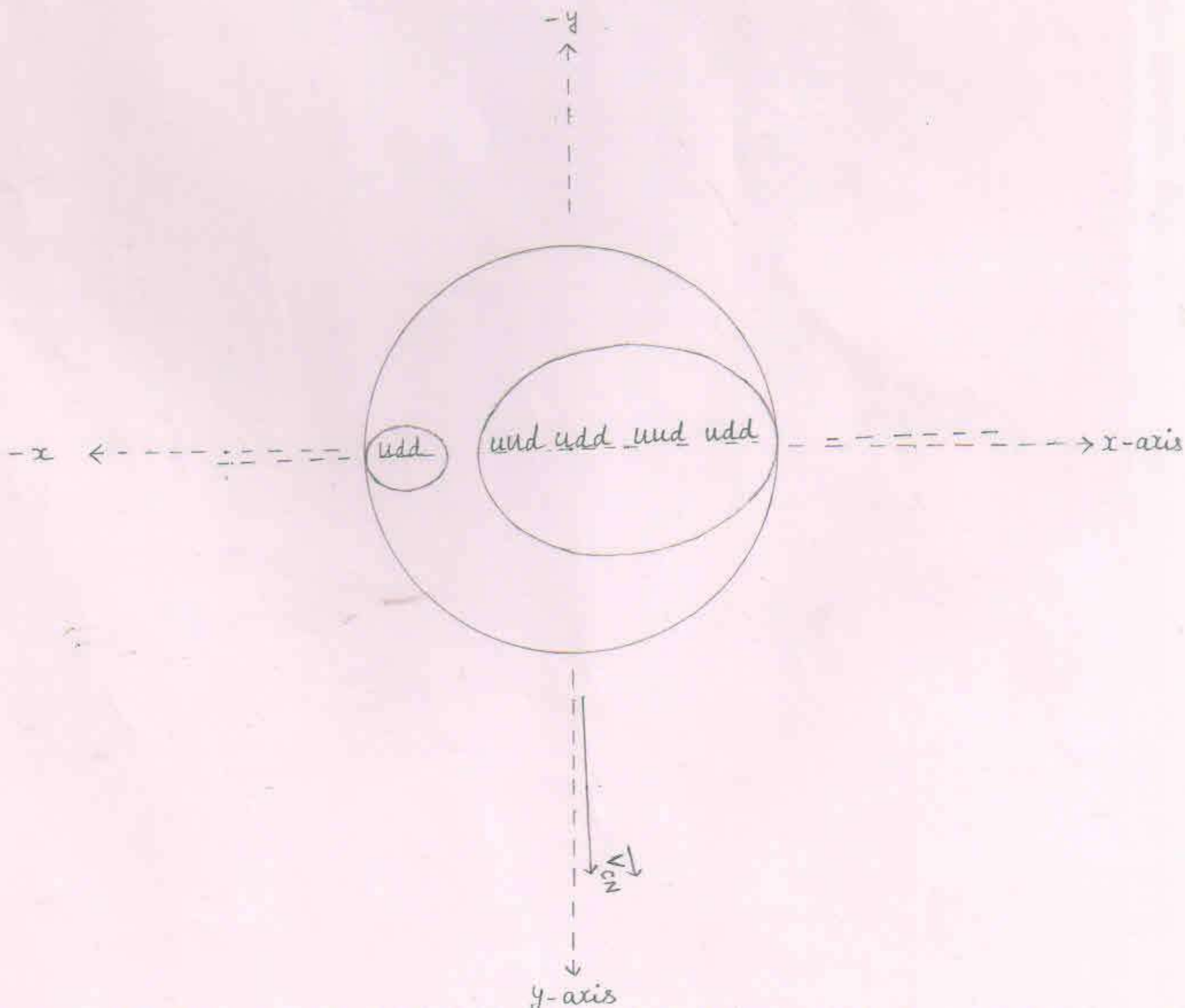
While the remaining group of quarks to become a stable nucleus (the neutron) includes its surrounding gluons or mass [out of the available mass (or gluons) that is not included in the formation of the lobe 'A'] and rearrange to form the 'B' lobe of the heterogenous compound nucleus.

Thus due to formation of two dissimilar lobes within into the homogenous compound nucleus, the homogenous compound nucleus transforms into the heterogenous compound nucleus.

Formation of lobes

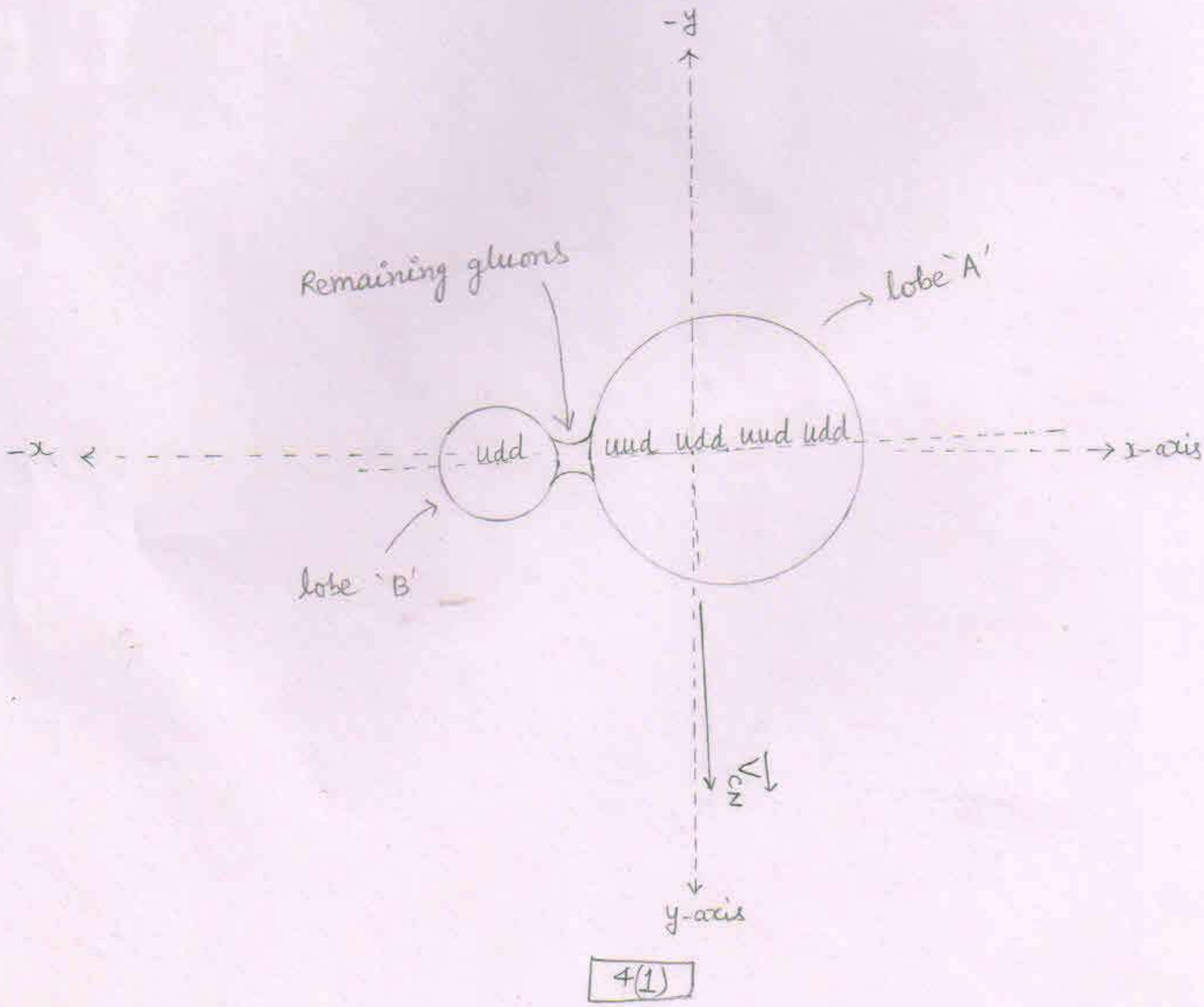
⇒ Within into the homogenous compound nucleus, the greater nucleus is the helium-4 nucleus and the smaller nucleus is the neutron while the remaining space represents the remaining gluons.

⇒ Within into the homogenous compound nucleus, the greater nucleus is the lobe 'A' while the smaller one is the lobe 'B'.

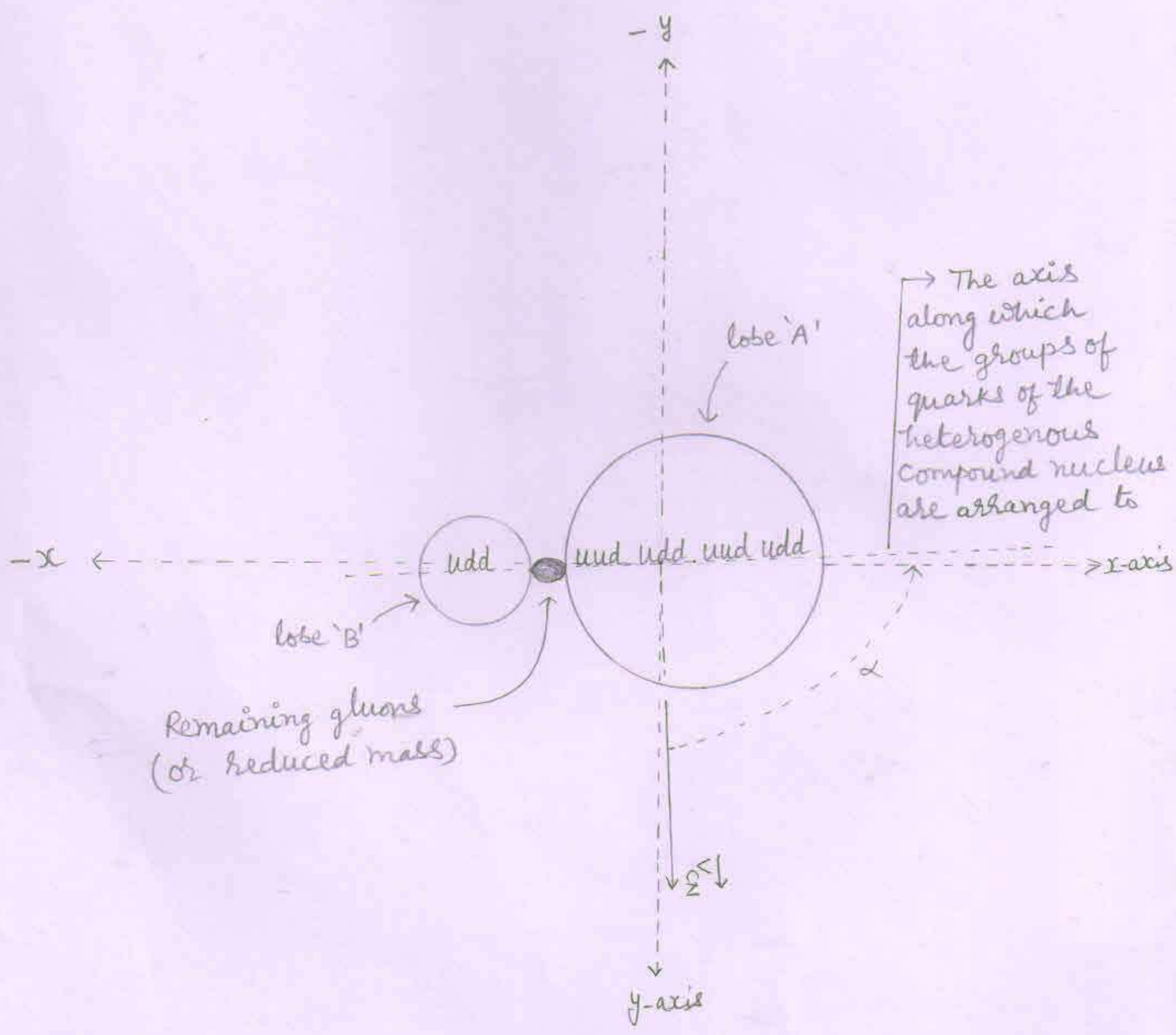


4. Final stage of the heterogenous compound nucleus:-
The process of formation of lobes creates voids between the lobes. So, the remaining gluons [or the mass that is not involved in the formation of any lobe] rearrange to fill the void(s) between the lobes and thus the remaining gluons form a node between the two dissimilar lobes of the heterogenous compound nucleus.
Thus the reduced mass (or the remaining gluons) keeps both the dissimilar lobes - of the heterogenous compound nucleus - joined them together.
So, finally, the heterogenous compound nucleus becomes like an abnormal digit eight or becomes like a dumbbell.

The heterogenous compound nucleus



Final stage of the heterogenous compound nucleus



Formation of Compound nucleus

1. As the deuteron of n bunch reaches at point F, it fuses with the confined triton to form a compound nucleus.

2. Just before fusion, to overcome the electrostatic repulsive force exerted by the triton, the deuteron of bunch loses its energy equal to 5.0622 kev.

So, just before fusion, the kinetic energy of deuteron is—

$$E_b = 102.4 \text{ kev} - 5.0622 \text{ kev}$$
$$= 97.3378 \text{ kev}$$
$$= 0.0973378 \text{ Mev}$$

3. Velocity of n^{th} deuteron just before fusion:

$$E_b = \frac{1}{2} m_d v^2 = 0.0973378 \text{ Mev}$$

$$v = \left[\frac{2 \times 0.0973378 \times 1.6 \times 10^{-13} \text{ J}}{3.3434 \times 10^{-27} \text{ kg}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{0.31148096 \times 10^{-13}}{3.3434 \times 10^{-27}}\right]^{\frac{1}{2}} \text{ m/s}$$

$$= [0.0931629359 \times 10^{14}]^{\frac{1}{2}} \text{ m/s}$$

$$= 0.3052 \times 10^{7} \text{ m/s}$$

Components of velocity of deuteron (just before fusion) at point F

$\Rightarrow$ As the deuteron is injected making angle 30° with x-axis, 60° angle with y-axis, and 90° angle with z-axis.

So, just before fusion, the components of velocity of injected deuteron at point F are –

1. $\vec{v}_x = v \cos\alpha$

$\cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\vec{v}_x = 0.3052 \times 10^7 \times \frac{1.732}{2} \text{ m/s}$$

$$= 0.2643 \times 10^7 \text{ m/s}$$

2. $\vec{v}_y = v \cos\beta$

$v = 0.3052 \times 10^7$ m/s

$\cos\beta = \cos 60^\circ = \frac{1}{2}$

$$\vec{v}_y = 0.3052 \times 10^7 \times \frac{1}{2} \text{ m/s}$$

$$= 0.1526 \times 10^7 \text{ m/s}$$

3. $\vec{v}_z = v\cos\gamma = v\cos 90^\circ = v \times 0 = 0$ m/s

Components of momentum of deuteron (just before fusion) at point F

1. $\vec{p}_x = m_d \vec{v}_x$

$m_d = 3.3434 \times 10^{-27}$ kg

$v_x = 0.2643 \times 10^{7}$ m/s

$\vec{p}_x = 3.3434 \times 10^{-27} \times 0.2643 \times 10^{7}$ kg m/s

$= 0.8836 \times 10^{-20}$ kg m/s

2. $\vec{p}_y = m_d \vec{v}_y$

$\vec{v}_y = 0.1526 \times 10^{7}$ m/s

$\vec{p}_y = 3.3434 \times 10^{-27} \times 0.1526 \times 10^{7}$ kg m/s

$= 0.5102 \times 10^{-20}$ kg m/s

3. $\vec{p}_z = m_d \vec{v}_z$

$\vec{v}_z = 0$ m/s

$= m_d \times 0$ kg m/s

$= 0$ kg m/s

Components of momentum of Compound nucleus

1. X-component of momentum of compound nucleus =

$$\begin{bmatrix} \text{x-component} \\ \text{of momentum of} \\ \text{confined triton} \\ \text{at point F} \end{bmatrix} + \begin{bmatrix} \text{x-component of momentum} \\ \text{of injected deuteron} \\ \text{(just before fusion)} \\ \text{at point F} \end{bmatrix}$$

$$\vec{P}_x = [-0.7005 \times 10^{-20}\, kgm/s] + [0.8836 \times 10^{-20}\, kgm/s]$$

$$= 0.1831 \times 10^{-20}\ kgm/s$$

2. y-component of momentum of compound nucleus =

$$\begin{bmatrix} \text{y-component} \\ \text{of momentum of} \\ \text{confined triton} \\ \text{at point F} \end{bmatrix} + \begin{bmatrix} \text{y-component of momentum} \\ \text{of injected deuteron} \\ \text{(just before fusion)} \\ \text{at point F} \end{bmatrix}$$

$$\vec{P}_y = [4.2556 \times 10^{-20}\, kgm/s] + [0.5102 \times 10^{-20}\, kgm/s]$$

$$= 4.7658 \times 10^{-20}\ kgm/s$$

3. Z-Component of momentum of Compound nucleus =

$$\begin{bmatrix} \text{z-Component} \\ \text{of momentum} \\ \text{of confined triton} \\ \text{at point F} \end{bmatrix} + \begin{bmatrix} \text{Z-Component of momentum} \\ \text{of injected deuteron} \\ \text{(just before fusion)} \\ \text{at point F} \end{bmatrix}$$

$$\vec{P}_z = 0 \text{ kg m/s} + 0 \text{ kg m/s}$$

$$= 0 \text{ kg m/s}$$

4. Mass of the Compound nucleus

$$M = [m_d + m_t]$$

$$= [3.3434 \times 10^{-27} \text{ kg} + 5.0072 \times 10^{-27} \text{ kg}]$$

$$= 8.3506 \times 10^{-27} \text{ kg}$$

Components of velocity of compound nucleus

1. $\vec{v}_x = v_{CN} \cos\alpha = \dfrac{\vec{P}_x}{M}$

$= \dfrac{0.1831 \times 10^{-20}}{8.3506 \times 10^{-27}} \dfrac{kg\,m/s}{kg}$

$= 0.0219 \times 10^{7}$ m/s

2. $\vec{v}_y = v_{CN} \cos\beta = \dfrac{\vec{P}_y}{M}$

$= \dfrac{4.7658 \times 10^{-20}}{8.3506 \times 10^{-27}} \dfrac{kg\,m/s}{kg}$

$= 0.5707 \times 10^{7}$ m/s

3. $\vec{v}_z = v_{CN} \cos\gamma = \dfrac{\vec{P}_z}{M} = \dfrac{0}{M} = 0$ m/s

4. $v_{CN}^2 = v_x^2 + v_y^2 + v_z^2$

$= (0.0219 \times 10^{7})^2 + (0.5707 \times 10^{7})^2 + (0)^2 \; m^2/s^2$

$= (0.00047961 \times 10^{14}) + (0.32569849 \times 10^{14}) + 0 \; m^2/s^2$

$v_{CN}^2 = 0.3261781 \times 10^{14} \; m^2/s^2$

$v_{CN} = 0.5711 \times 10^{7}$ m/s

Angles that make the velocity of compound nucleus ($\vec{V}_{CN}$) with axes :-

1. With x-axis

$$\cos\alpha = \frac{V_{CN}\cos\alpha}{V_{CN}} = \frac{\vec{V_x}}{V_{CN}} = \frac{0.0219\times10^7}{0.5711\times10^7}\ \frac{m/s}{m/s}$$

$\Rightarrow \cos\alpha = 0.0383$

$\Rightarrow \alpha \simeq 87.8$

2. With y-axis

$$\cos\beta = \frac{V_{CN}\cos\beta}{V_{CN}} = \frac{\vec{V_y}}{V_{CN}} = \frac{0.5707\times10^7}{0.5711\times10^7}\ \frac{m/s}{m/s}$$

$\Rightarrow \cos\beta = 0.9992$

$\beta = 2.2$

3. With z-axis

$$\cos\gamma = \frac{V_{CN}\cos\gamma}{V_{CN}} = \frac{\vec{V_z}}{V_{CN}} = \frac{0}{0.5711\times10^7}\ \frac{m/s}{m/s}$$

$\Rightarrow \cos\gamma = 0$

$\Rightarrow \gamma = 90^\circ$

The Splitting of the heterogenous compound nucleus :-

⇒ The heterogenous compound nucleus, due to its instability, splits according to the lines parallel to the direction of the velocity of the compound nucleus ($\vec{v}_{CN}$) into three particles - the helion-4, the neutron and the reduced mass (Δm).

out of them, the two particles (the helion-4 and the neutron) are stable while the third one (reduced mass) is unstable.

⇒ According to the law of inertia, each particle that has separated from the compound nucleus, has an inhereted velocity ($\vec{v}_{inh}$) equal to the velocity of the compound nucleus ($\vec{v}_{CN}$).

⇒ So, for conservation of momentum

$$M\vec{V}_{CN} = (m_{he-4} + \Delta m + m_{n})\vec{V}_{CN}$$

Where,

M = mass of the compound nucleus

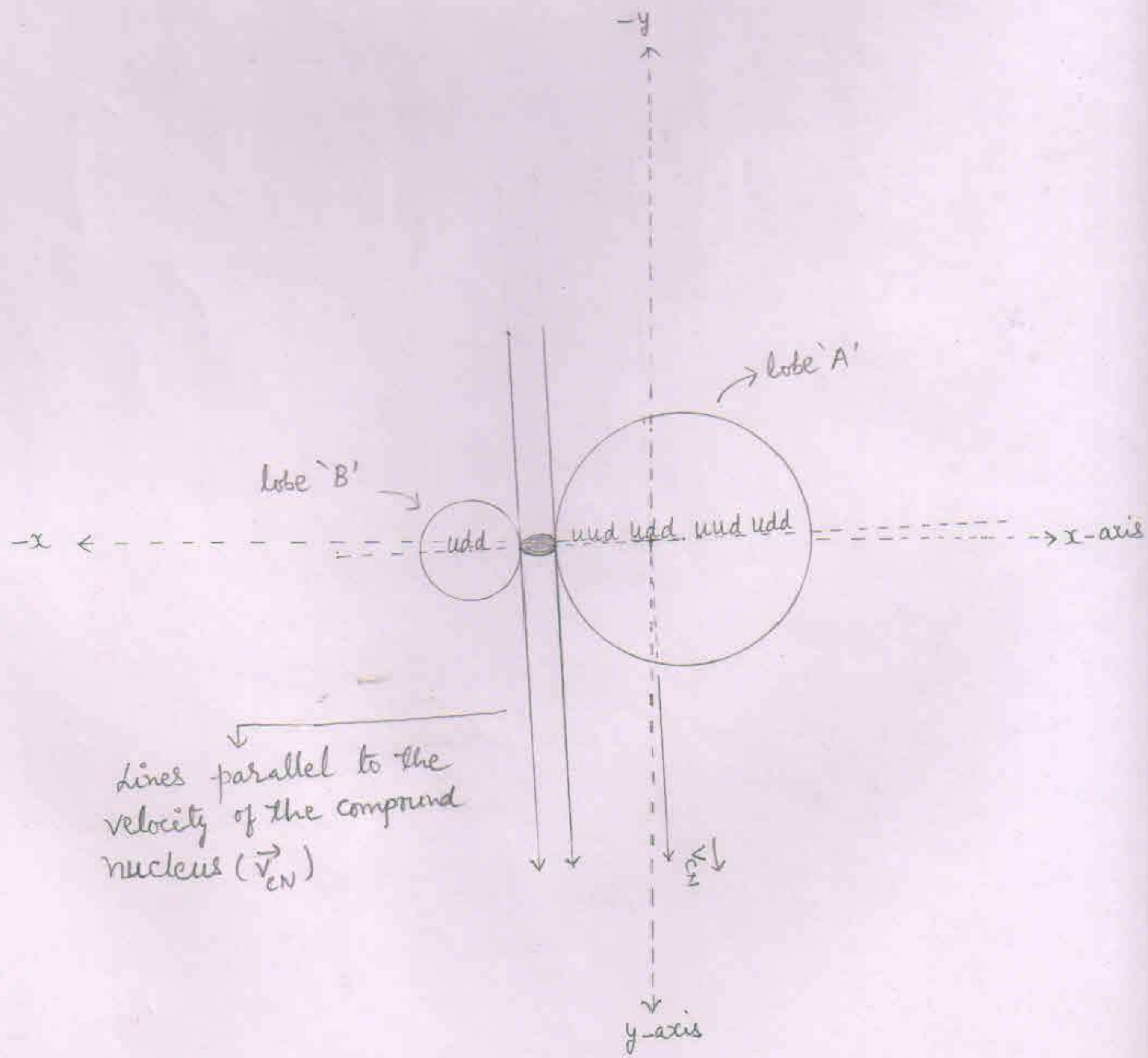
$\vec{V}_{CN}$ = velocity of the compound nucleus

Δm = reduced mass

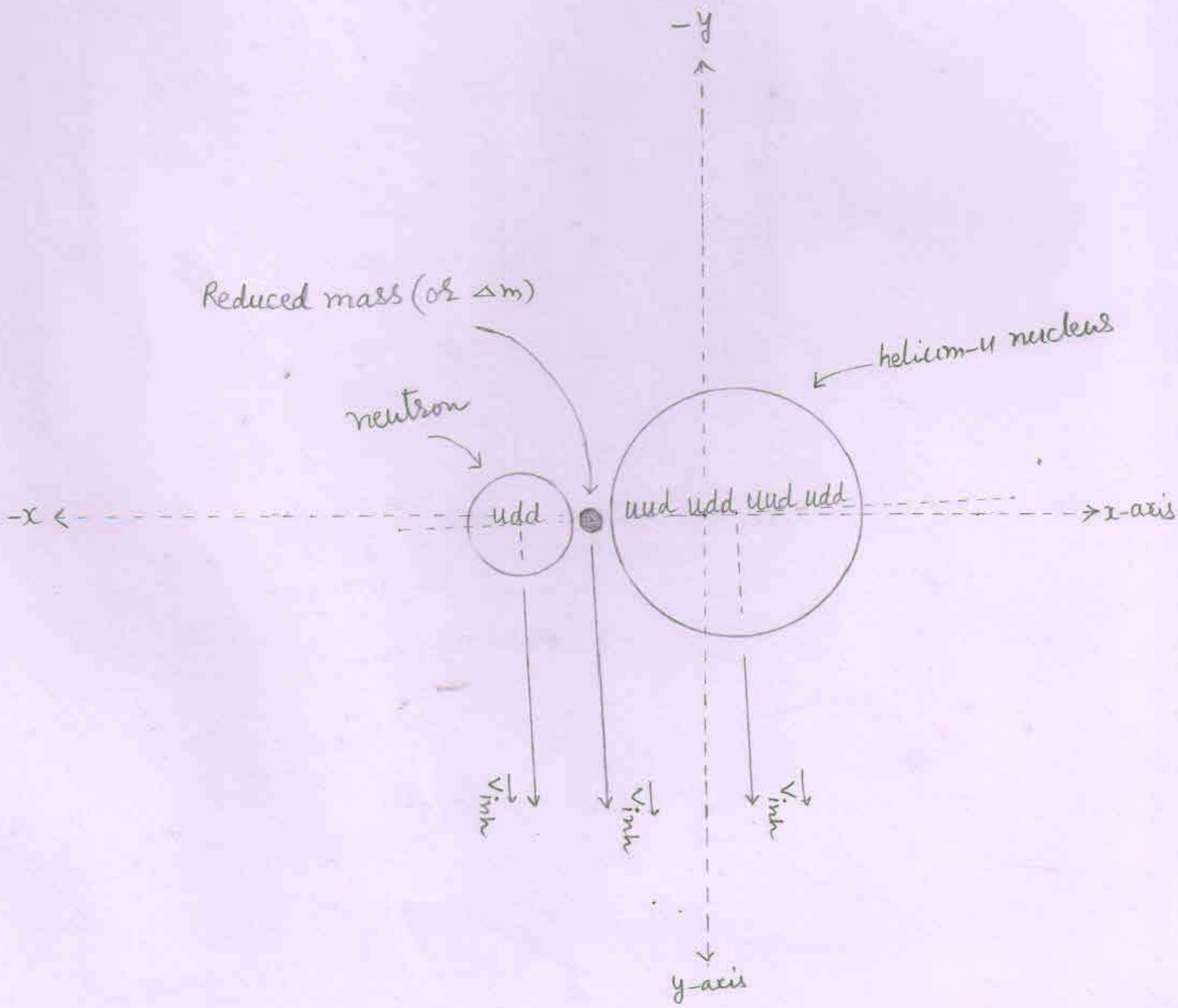
m_{he-4} = mass of the helium-4 nucleus

m_{n} = mass of the neutron

The splitting of the heterogenous compound nucleus



The splitting of the heterogenous compound nucleus



Inhereted Velocity of the particle

$\Rightarrow$ Each particle has an inhereted velocity ($\vec{v}_{inh}$) equal to the velocity of compound nucleus ($\vec{V}_{CN}$)

I. Inhereted velocity of the particle ${}_2He^4$

$$v_{inh} = v_{CN} = 0.5711 \times 10^7 \ m/s$$

$\Rightarrow$ Components of inhereted velocity of particle He-4

1. $\vec{v}_x = v_{inh} \cos\alpha = v_{CN} \cos\alpha = 0.0219 \times 10^7 \ m/s$

2. $\vec{v}_y = v_{inh} \cos\beta = v_{CN} \cos\beta = 0.5707 \times 10^7 \ m/s$

3. $\vec{v}_z = v_{inh} \cos\gamma = v_{CN} \cos\gamma = 0 \ m/s$

II. Inhereted velocity of the particle - neutron

1. $v_{inh} = v_{CN} = 0.5711 \times 10^7 \ m/s$

$\Rightarrow$ Components of inhereted velocity of neutron

1. $\vec{v}_x = v_{inh} \cos\alpha = v_{CN} \cos\alpha = 0.0219 \times 10^7 \ m/s$

2. $\vec{v}_y = v_{inh} \cos\beta = v_{CN} \cos\beta = 0.5707 \times 10^7 \ m/s$

3. $\vec{v}_z = v_{inh} \cos\gamma = v_{CN} \cos\gamma = 0 \ m/s$

III. Inhereted velocity of reduced mass

1. $v_{inh} = v_{CN} = 0.5711 \times 10^7 \ m/s$

Propellation of particles

Reduced mass converts into energy and the total energy (E_T) propell both the particles with equal and opposite momentum.

1. Reduced mass

$$\Delta m = [m_t + m_d] - [m_{He-4} + m_n]$$

$$= [3.0155 + 2.01355] - [4.0015 + 1.00866] \text{ amu}$$

$$= [5.02905] - [5.01016] \text{ amu}$$

$$= 0.01889 \text{ amu}$$

$$= 0.01889 \times 1.6605 \times 10^{-27} \text{ kg}$$

$$= 0.031366 \times 10^{-27} \text{ kg}$$

2. Inhereted kinetic energy of reduced mass

$$E_{inh} = \frac{1}{2} \Delta m V_{CN}^2$$

$$= \frac{1}{2} \times 0.031366 \times 10^{-27} \times 0.3261781 \times 10^{14} \text{ J}$$

$$= 0.00511545114 \times 10^{-13} \text{ J}$$

$$= 0.0031 \text{ MeV}$$

3. Released energy (E_R) :

$$E_R = \Delta m c^2$$
$$= 0.01889 \times 931 \text{ Mev}$$

$$= 17.58659 \text{ Mev}$$

4. Total energy

$$E_T = E_{inh} + E_R$$

$$= [0.0031] + [17.58659] \text{ Mev}$$

$$= 17.58969 \text{ Mev}$$

Increased energy of the particle

The total energy E_T is divided between the particles according to their inverse masses. So, the increased energy (E_{inc}) of the particle

1. Increased energy of the He-4

$$E_{inc} = \frac{m_n}{m_n + m_{He-4}} \times E_T$$

$$= \frac{1.00866 \text{ amu}}{[1.00866 + 4.0015] \text{ amu}} \times 17.58969 \text{ Mev}$$

$$= \frac{1.00866}{5.01016} \times 17.58969 \text{ Mev}$$

$$= 0.20132291184 \times 17.58969 \text{ Mev}$$

$$= 3.5412 \text{ Mev}$$

2. Increased energy of the neutron

$$E_{inc} = [E_T] - [\text{Increased energy of the He-4}]$$

$$= [17.58969] - [3.5412] \text{ Mev}$$

$$= 14.04849 \text{ Mev}$$

Increased velocity of the particle

1. For He-4

$$V_{inc} = \left[\frac{2E_{inc}}{m_{He\text{-}4}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2\times 3.5412\times 1.6\times 10^{-13}\ J}{6.64449\times 10^{-27}\ kg}\right]^{\frac{1}{2}}$$

$$= \left[\frac{11.33184\times 10^{-13}}{6.64449\times 10^{-27}}\right]^{\frac{1}{2}}\ m/s$$

$$= \left[1.70544917668\times 10^{14}\right]^{\frac{1}{2}}\ m/s$$

$$= 1.3059\times 10^{7}\ m/s$$

2. For neutron

$$V_{inc} = \left[\frac{2E_{inc}}{m_n}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2\times 14.04849\times 1.6\times 10^{-13}\ J}{1.6749\times 10^{-27}\ kg}\right]$$

$$= \left[\frac{44.955168\times 10^{-13}}{1.6749\times 10^{-27}}\right]^{\frac{1}{2}}\ m/s$$

$$= \left[26.840508687\times 10^{14}\right]^{\frac{1}{2}}\ m/s$$

Angle of propellation

1. As the reduced mass converts into energy, the total energy (E_T) propell both the particles with equal and opposite momentum according to a ray (line) perpendicular to the direction (line) of the velocity of the compound nucleus ($\vec{V}_{CN}$).

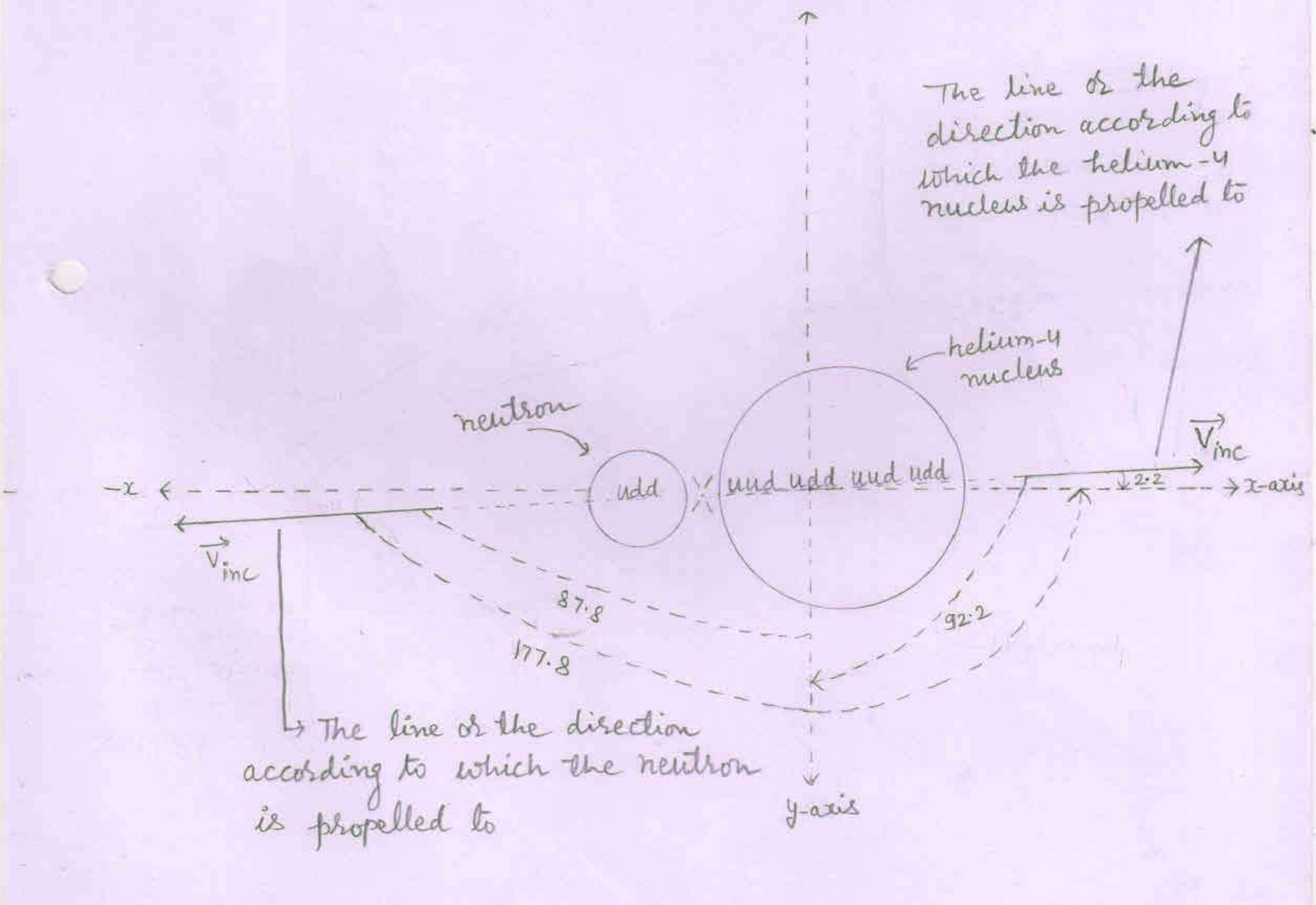
2. We know that when there a fusion process occures, then we find the lighter nucleus in the forward direction [or in the direction of ion beam or in the direction of the velocity of the compound nucleus ($\vec{V}_{CN}$).]

3. At point `F', as $\vec{V}_{CN}$ makes 87.8 degree angle with x-axis, 2.2 degree angle with y-axis and 90° angle with z-axis.

4. So, the neutron is propelled making 177.8 degree angle with x-axis, 87.8 degree angle with y-axis and 90° angle with z-axis.

5. While the helion-4 is propelled making 2.2 degree angle with x-axis, 92.2 degree angle with y-axis and 90° angle with z-axis.

Propellation of the particles



Components of the increased velocity of the particles

I For neutron

1. $\vec{V_x} = V_{inc} \cos\alpha$

$V_{inc} = 5.1807 \times 10^7$ m/s

$\cos\alpha = \cos(177.8) = -\cos(2.2) = -0.99$

$\Rightarrow \vec{V_x} = 5.1807 \times 10^7 \times (-0.99)$ m/s
$= -5.1288 \times 10^7$ m/s

2. $\vec{V_y} = V_{inc} \cos\beta$

$\cos\beta = \cos(87.8) = 0.03$

$\Rightarrow \vec{V_y} = 5.1807 \times 10^7 \times 0.03$ m/s
$= 0.1554 \times 10^7$ m/s

3. $\vec{V_z} = V_{inc} \cos\gamma$

$\cos\gamma = \cos 90^\circ = 0$

$\Rightarrow \vec{V_z} = 5.1807 \times 10^7 \times 0$ m/s $= 0$ m/s

II For helion-4

1. $\vec{V_x} = V_{inc} \cos\alpha$

$\cos\alpha = \cos(2.2) = 0.99$

$V_{inc} = 1.3059 \times 10^7$ m/s

$\Rightarrow \vec{V_x} = 1.3059 \times 10^7 \times 0.99$ m/s
$= 1.2928 \times 10^7$ m/s

2. $\vec{V_y} = V_{inc} \cos\beta$

$\cos\beta = \cos(92.2) = -\cos(87.8) = -0.03$

$\Rightarrow \vec{V_y} = 1.3059 \times 10^7 \times (-0.03)$ m/s
$= -0.0391 \times 10^7$ m/s

3. $\vec{V_z} = V_{inc} \cos\gamma$

$\cos\gamma = \cos 90^\circ = 0$

$\Rightarrow \vec{V_z} = 1.3059 \times 10^7 \times 0$ m/s $= 0$ m/s

Components of the final velocity ($\vec{v}_f$) of the particles

I. For neutron

According to	Inhereted velocity ($\vec{v}_{inh}$)	Increased velocity ($\vec{v}_{inc}$)	Final velocity ($\vec{v}_f$) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = 0.0219 \times 10^7$ m/s	$\vec{v}_x = -5.1288 \times 10^7$ m/s	$\vec{v}_x = -5.1069 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.5707 \times 10^7$ m/s	$\vec{v}_y = 0.1554 \times 10^7$ m/s	$\vec{v}_y = 0.7261 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

II For helium-4 nucleus

According to	Inhereted velocity ($\vec{v}_{inh}$)	Increased velocity ($\vec{v}_{inc}$)	Final velocity ($\vec{v}_f$) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = 0.0219 \times 10^7$ m/s	$\vec{v}_x = 1.2928 \times 10^7$ m/s	$\vec{v}_x = 1.3147 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.5707 \times 10^7$ m/s	$\vec{v}_y = -0.0391 \times 10^7$ m/s	$\vec{v}_y = 0.5316 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

Final velocity of the neutron

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$v_x = 5.1069 \times 10^7$ m/s
$v_y = 0.7261 \times 10^7$ m/s
$v_z = 0$ m/s

$$\Rightarrow v_f^2 = (5.1069 \times 10^7)^2 + (0.7261 \times 10^7)^2 + (0)^2 \quad m^2/s^2$$

$$\Rightarrow v_f^2 = (26.08042761 \times 10^{14}) + (0.52722121 \times 10^{14}) + 0 \quad m^2/s^2$$

$$\Rightarrow v_f^2 = 26.60764882 \times 10^{14} \quad m^2/s^2$$

$$\Rightarrow v_f = 5.1582 \times 10^7 \quad m/s$$

Final kinetic energy of the neutron

$$E = \frac{1}{2} m_n v_f^2$$

$v_f^2 = 26.60764882 \times 10^{14}$ m^2/s^2

$$\Rightarrow E = \frac{1}{2} \times 1.6749 \times 10^{-27} \times 26.60764882 \times 10^{14} \text{ J}$$

$$= 22.2825755043 \times 10^{-13} \text{ J}$$

$$= 13.9266 \text{ MeV}$$

Angle made by the final velocity ($\vec{V_f}$) of the neutron with respect to axes at point F.

1. With x-axis

$$\cos\alpha = \frac{\vec{V_x}}{V_f}$$

$$\vec{V_x} = -5.1069 \times 10^7 \text{ m/s}$$
$$V_f = 5.1582 \times 10^7 \text{ m/s}$$

$$\Rightarrow \cos\alpha = \frac{-5.1069 \times 10^7}{5.1582 \times 10^7} \frac{\text{m/s}}{\text{m/s}} = -0.9900$$

$$\Rightarrow \alpha \simeq 171.9 \text{ degree} \quad [\because \cos(171.9) = -0.9900]$$

2. With y-axis

$$\cos\beta = \frac{\vec{V_y}}{V_f}$$

$$\vec{V_y} = 0.7261 \times 10^7 \text{ m/s}$$

$$\Rightarrow \cos\beta = \frac{0.7261 \times 10^7}{5.1582 \times 10^7} \frac{\text{m/s}}{\text{m/s}} = 0.1407$$

$$\Rightarrow \beta \simeq 81.9 \text{ degree} \quad [\because \cos(81.9) = 0.1409]$$

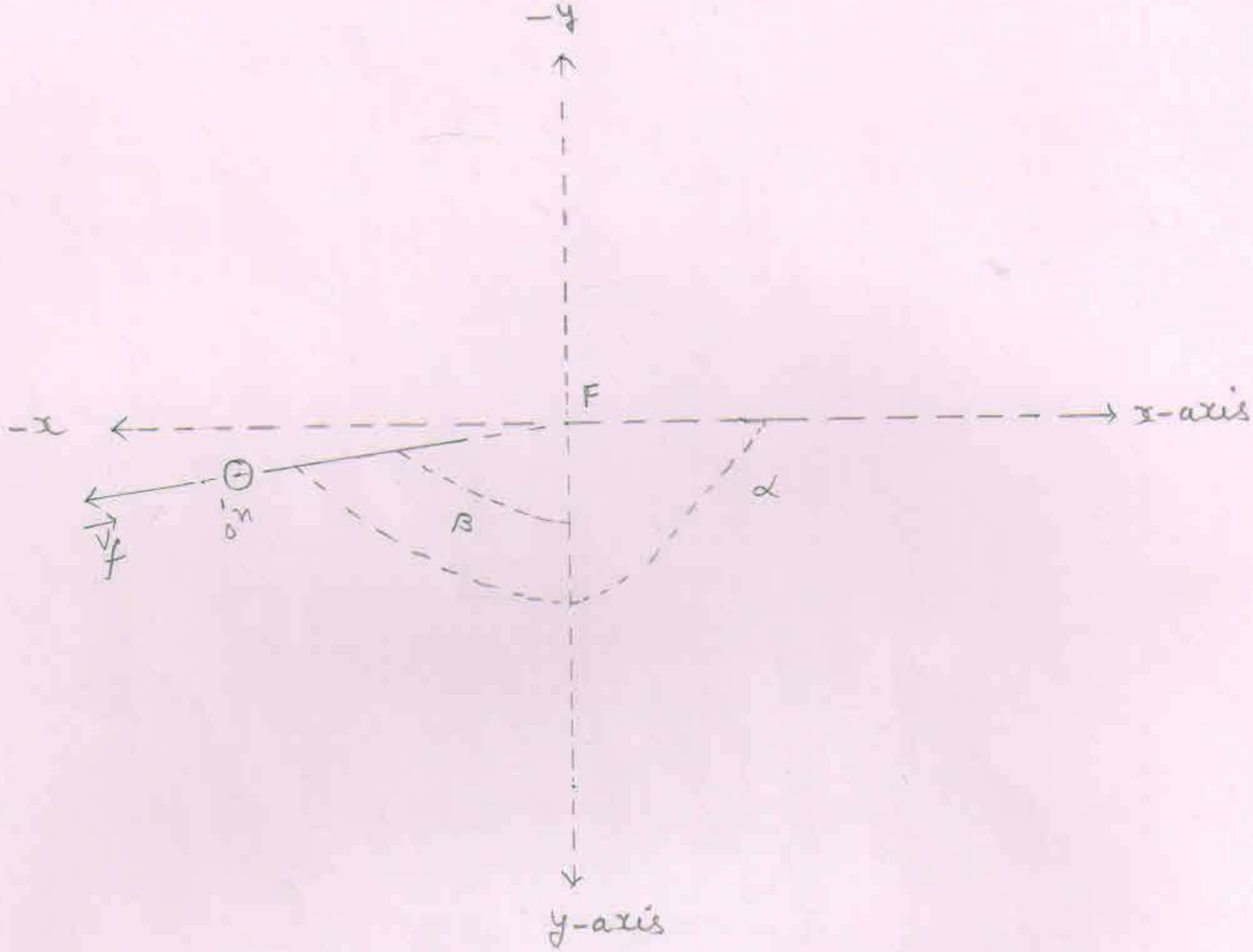
3. With z-axis

$$\cos\gamma = \frac{\vec{V_z}}{V_f}$$

$$\vec{V_z} = 0 \text{ m/s}$$

$$\Rightarrow \cos\gamma = \frac{0}{5.1582 \times 10^7} \frac{\text{m/s}}{\text{m/s}} = 0$$

$$\Rightarrow \gamma = 90^\circ$$



⇒ At point F, the final velocity ($\vec{v}_f$) of the neutron make angle α, β and γ with positive x, y and z axes respectively.

Where,

$$\alpha \simeq 171.9 \text{ degree}$$

$$\beta \simeq 81.9 \text{ degree}$$

$$\gamma = 90^\circ$$

Final velocity of the helion-4

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$v_x = 1.3147 \times 10^7$ m/s
$v_y = 0.5316 \times 10^7$ m/s
$v_z = 0$ m/s

$$\Rightarrow v_f^2 = (1.3147 \times 10^7)^2 + (0.5316 \times 10^7)^2 + (0)^2 \ m^2/s^2$$

$$\Rightarrow v_f^2 = (1.72843609 \times 10^{14}) + (0.28259856 \times 10^{14}) + 0 \ m^2/s^2$$

$$\Rightarrow v_f^2 = 2.01103465 \times 10^{14} \ m^2/s^2$$

$$\Rightarrow v_f = 1.4187 \times 10^7 \ m/s$$

Final kinetic energy of the helion-4

$$E = \frac{1}{2} m_{He-4} v_f^2$$

$v_f^2 = 2.01103465 \times 10^{14}$ m²/s²

$$\Rightarrow E = \frac{1}{2} \times 6.64449 \times 10^{-27} \times 2.01103465 \times 10^{14} \ J$$

$$= 6.68114981075 \times 10^{-13} \ J$$
$$= 4.1757 \ MeV$$

$$\Rightarrow m_{He-4} v_f^2 = 6.64449 \times 10^{-27} \times 2.01103465 \times 10^{14} \ J$$
$$= 13.3622 \times 10^{-13} \ J$$

Acting forces on the helium-4 nucleus

1. $F_y = q V_x B_z \sin\theta$

$\vec{V}_x = 1.3147 \times 10^7$ m/s

$\vec{B}_z = -1$ Tesla

$q = 2 \times 1.6 \times 10^{-19}$ C

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_y = 2 \times 1.6 \times 10^{-19} \times 1.3147 \times 10^7 \times 1 \times 1$ N

$= 4.2070 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_y$ is according to $-y$ axis. So,

$$\vec{F}_y = -4.2070 \times 10^{-12} \text{ N}$$

2. $F_z = q V_x B_y \sin\theta$

$\vec{B}_y = 1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_z = 2 \times 1.6 \times 10^{-19} \times 1.3147 \times 10^7 \times 1 \times 1$ N

$= 4.2070 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_z$ is according to $-z$ axis. So,

$$\vec{F}_z = -4.2070 \times 10^{-12} \text{ N}$$

3. $F_x = q V_y B_z \sin\theta$

$\vec{V}_y = 0.5316 \times 10^7$ m/s

$\vec{B}_z = -1$ Tesla

$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_x = 2 \times 1.6 \times 10^{-19} \times 0.5316 \times 10^7 \times 1 \times 1$ N

$= 1.7011 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_x$ is according to $+x$ axis. So,

$$\vec{F}_x = 1.7011 \times 10^{-12} \text{ N}$$

4. Resultant force (F_R):

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$F_x = 1.7011 \times 10^{-12}$ N

$F = F_y = F_z = 4.2070 \times 10^{-12}$ N

$$\Rightarrow F_R^2 = F_x^2 + 2F^2$$

$$\Rightarrow F_R^2 = (1.7011 \times 10^{-12})^2 + 2(4.2070 \times 10^{-12})^2 - N^2$$

$$\Rightarrow F_R^2 = (2.89374121 \times 10^{-24}) + 2(17.698849 \times 10^{-24}) \ N^2$$

$$\Rightarrow F_R^2 = (2.89374121 \times 10^{-24}) + (35.397698 \times 10^{-24}) \ N^2$$

$$\Rightarrow F_R^2 = 38.29143921 \times 10^{-24} \ N^2$$

$$\Rightarrow F_R = 6.1880 \times 10^{-12} \ N$$

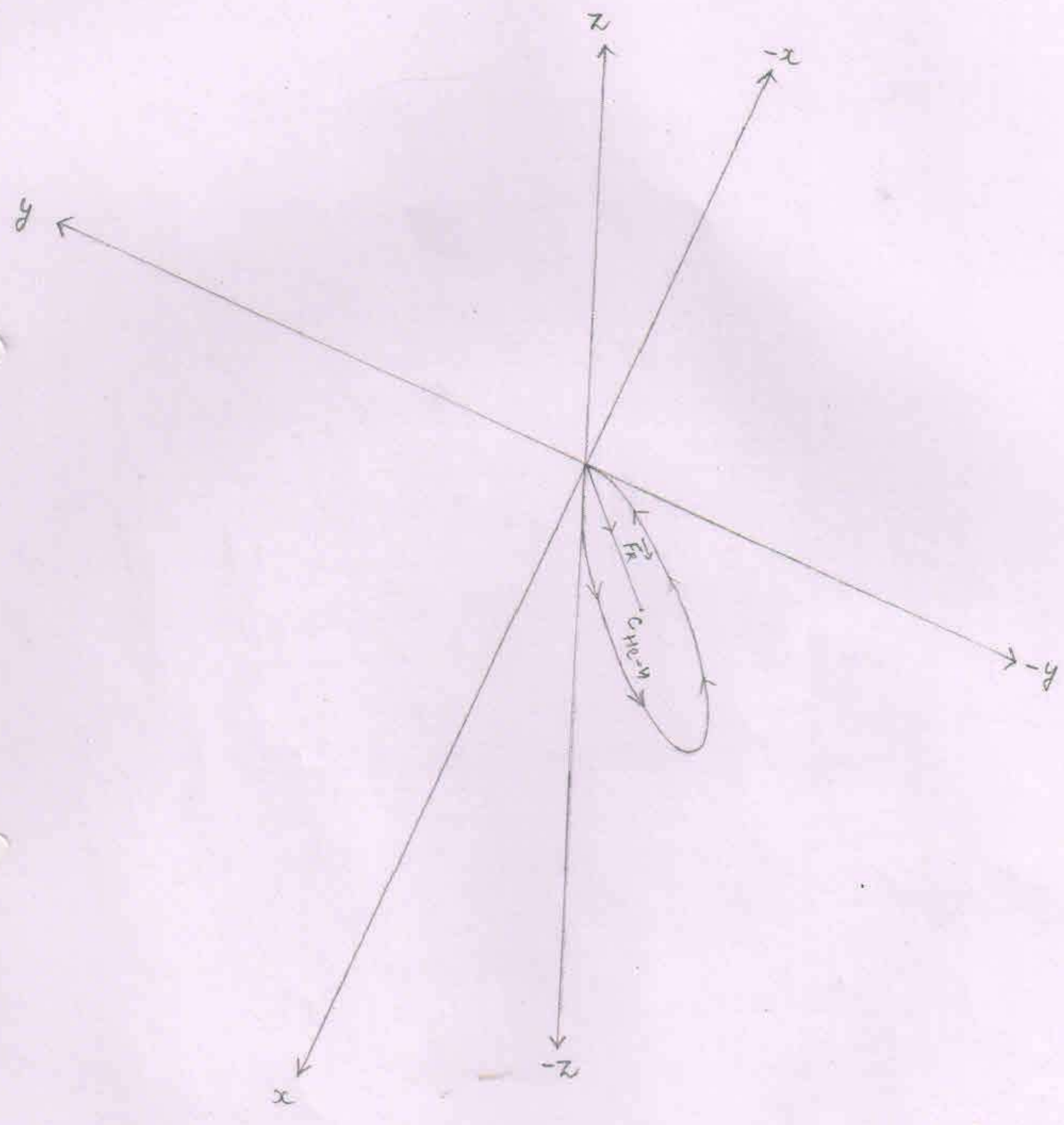
5. Radius of the circular orbit followed by the helion-4

$$r = \frac{mv^2}{F_R}$$

$$mv^2 = 13.3622 \times 10^{-13} \text{ J}$$
$$F_R = 6.1880 \times 10^{-12} \text{ N}$$

$$\Rightarrow r = \frac{13.3622 \times 10^{-13}}{6.1880 \times 10^{-12}} \frac{\text{J}}{\text{N}}$$

$$\Rightarrow r = 2.15937 \times 10^{-1} \text{ m}$$
$$= 21.5937 \times 10^{-2} \text{ m}$$



⇒ The circular orbit to be followed by the helion-4 lies in the IV (down) quadrant made up of positive x axis, negative y axis and the negative z axis.

⇒ C_{He-4} = centre of the circle to be followed by the helion-4

Angles that make the resultant force ($\vec{F}_R$) [acting on the helium-4 nucleus when the helium-4 nucleus is at point 'F'] with positive x, y and z-axes.

1. With x-axis

$$\cos\alpha = \frac{F_R \cos\alpha}{F_R} = \frac{\overrightarrow{F_x}}{F_R} = \frac{1.7011 \times 10^{-12}}{6.1880 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\alpha = 0.2749$

$\Rightarrow \alpha \simeq 74.05$ degree [$\because \cos(74.05) = 0.2747$]

2. With y-axis

$$\cos\beta = \frac{F_R \cos\beta}{F_R} = \frac{\overrightarrow{F_y}}{F_R} = \frac{-4.2070 \times 10^{-12}}{6.1880 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\beta = -0.6798$

$\Rightarrow \beta \simeq 132.8$ degree [$\because \cos(132.8) = -0.6794$]

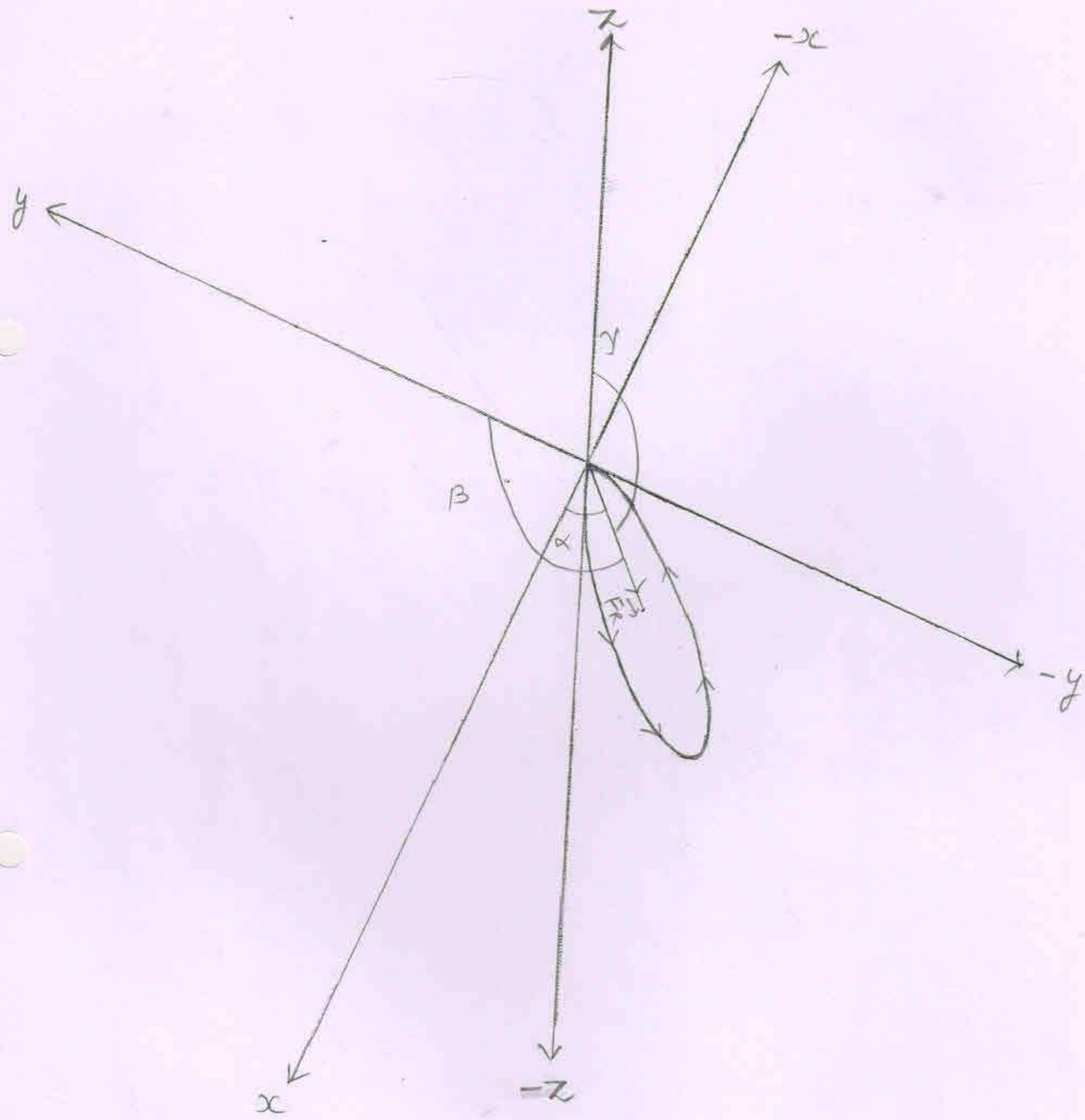
3. With z-axis

$$\cos\gamma = \frac{F_R \cos\gamma}{F_R} = \frac{\overrightarrow{F_z}}{F_R} = \frac{-4.2070 \times 10^{-12}}{6.1880 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\gamma = -0.6798$

$\Rightarrow \gamma \simeq 132.8$ degree

Angles that make the resultant force ($\vec{F_R}$) at point F with positive x, y and z axes.



where,

$\alpha \simeq 74.05$

$\beta \simeq 132.8$

$\gamma \simeq 132.8$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle oblained by the helium-4 nucleus :-

1. $\cos\alpha = \frac{x_2 - x_1}{d}$

$$d = 2 \times r$$
$$= 2 \times 21.5937 \times 10^{-2}\ m$$
$$= 43.1874 \times 10^{-2}\ m$$
$$\cos\alpha = 0.27$$

$\Rightarrow x_2 - x_1 = d \times \cos\alpha$

$\Rightarrow x_2 - x_1 = 43.1874 \times 10^{-2} \times 0.27\ m$

$\Rightarrow x_2 - x_1 = 11.6605 \times 10^{-2}\ m$

$\Rightarrow x_2 = 11.6605 \times 10^{-2}\ m \quad [\because x_1 = 0]$

2. $\cos\beta = \frac{y_2 - y_1}{d}$

$$\cos\beta = -0.67$$

$\Rightarrow y_2 - y_1 = d \times \cos\beta$

$\Rightarrow y_2 - y_1 = 43.1874 \times 10^{-2} \times (-0.67)\ m$

$\Rightarrow y_2 - y_1 = -28.9355 \times 10^{-2}\ m$

$\Rightarrow y_2 = -28.9355 \times 10^{-2}\ m \quad [\because y_1 = 0]$

3. $\cos\gamma = \frac{z_2 - z_1}{d}$

$$\cos\gamma = -0.67$$

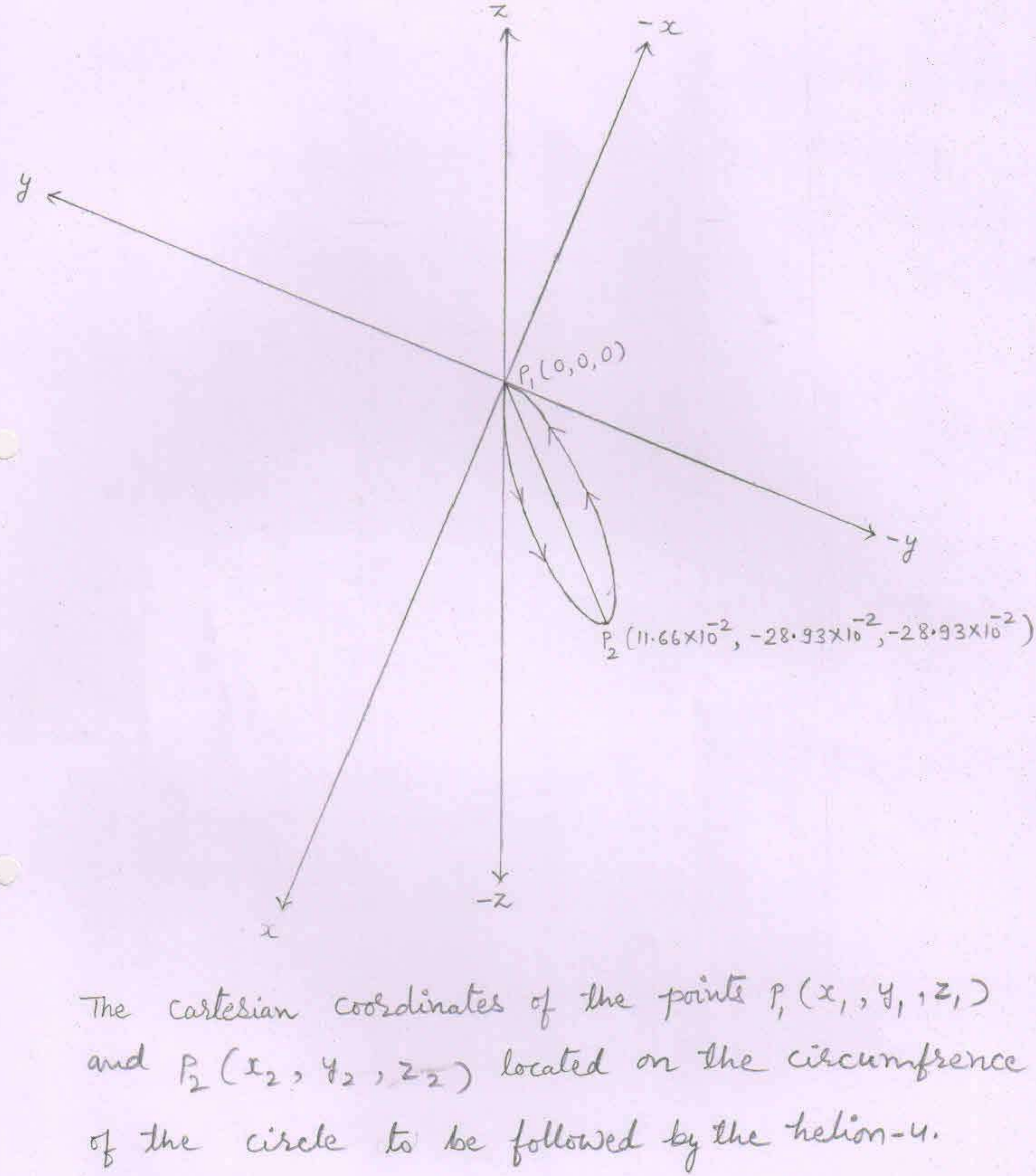
$\Rightarrow z_2 - z_1 = d \times \cos\gamma$

$\Rightarrow z_2 - z_1 = 43.1874 \times 10^{-2} \times (-0.67)\ m$

$\Rightarrow z_2 - z_1 = -28.9355 \times 10^{-2}\ m$

$\Rightarrow z_2 = -28.9355 \times 10^{-2}\ m \quad [\because z_1 = 0]$

The Cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$



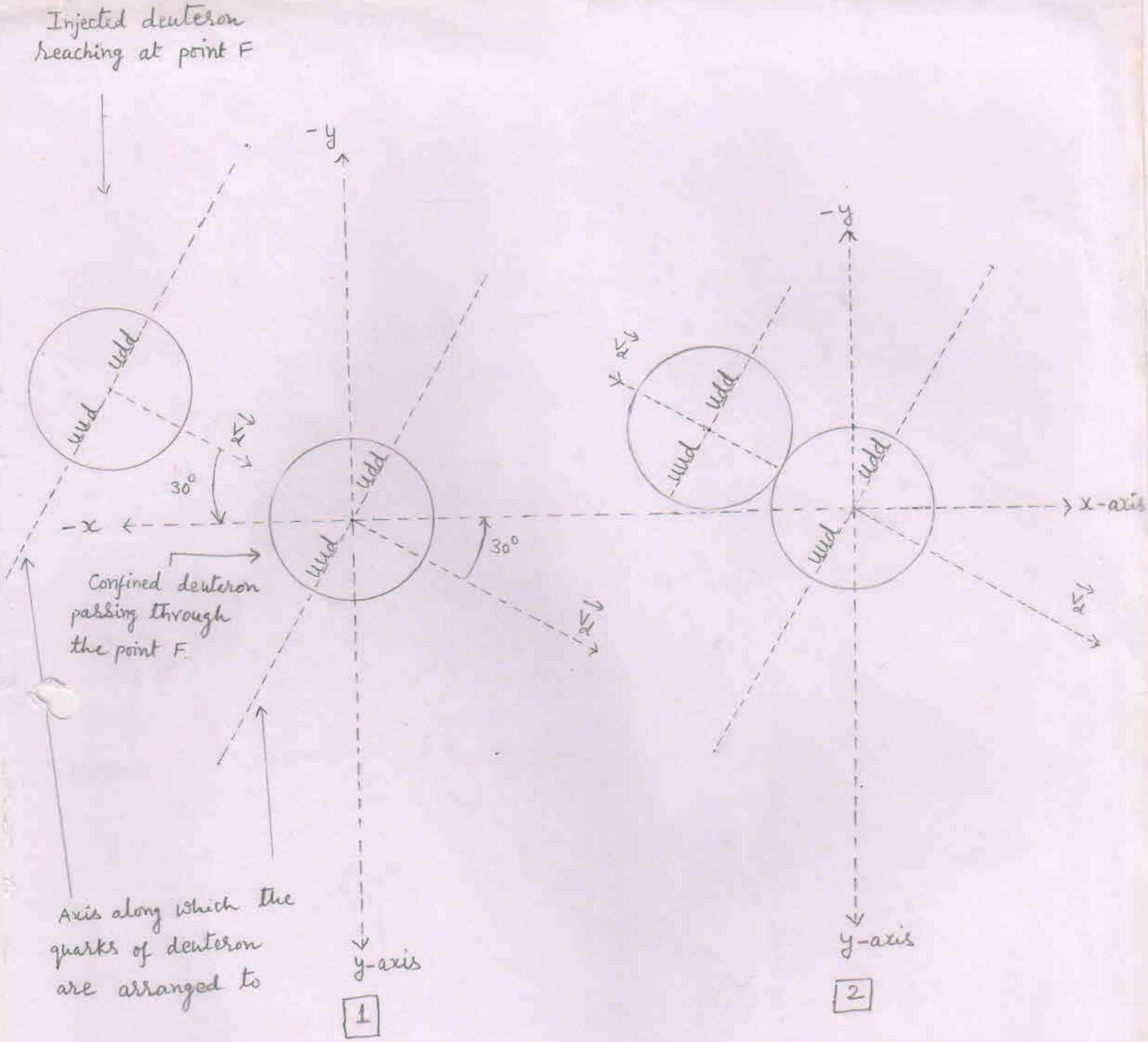
The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle to be followed by the helion-4.

$\exists$ The line $\overline{P_1P_2}$ is the diameter of the circle.

For fusion reaction : ${}^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{4}_{2}He + \gamma$ rays

1. Interaction of nuclei :-

The injected deuteron reaches at point 'F' and interacts [experiences a repulsive force due to confined deuteron] with the confined deuteron passing through the point 'F'. The injected deuteron overcomes the electrostatic repulsive force and - a like two solid spheres join - the injected deuteron dissimilarly joins with the confined deuteron.



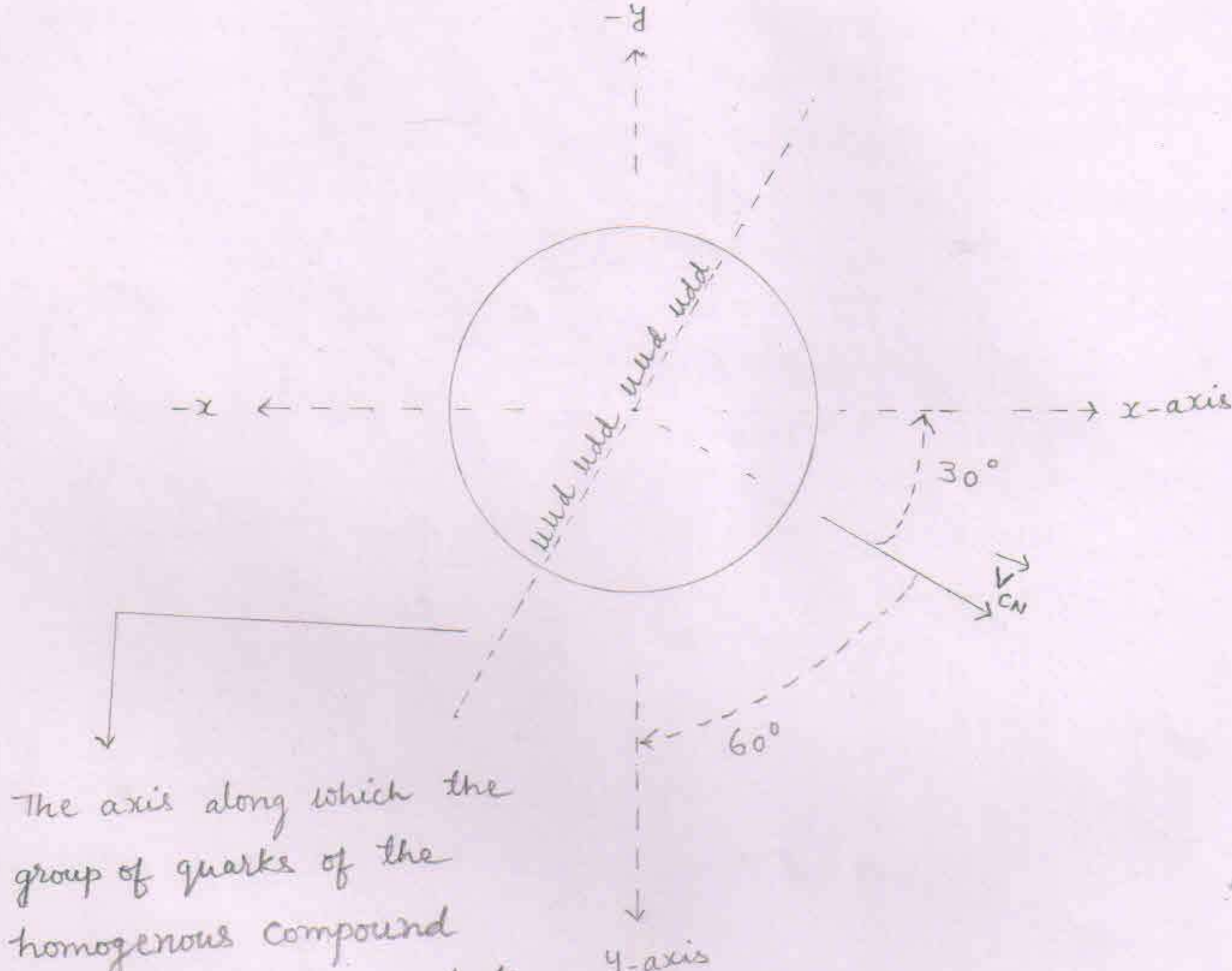
Injection of deuteron

Interaction of nuclei

2. Formation of homogenous compound nucleus :-
The constituents (quarks and gluons of the dissimilarly joined nuclei (deuterons) behave like a liquid and form a homogenous compound nucleus having similarly distributed groups of quarks with similarly distributed surrounding gluons.

Thus within the homogenous compound nucleus - each group of quarks is surrounded by the gluons in equal proportion. So, within the homogenous compound nucleus there are 4 groups of quarks with surrounding gluons.

The homogenous compound nucleus



$\Rightarrow$ V_{CN} = velocity of the compound nucleus

3. Formation of lobes within into the homogenous compound nucleus [${}^{4}_{2}M$] or the transformation of the homogenous compound nucleus into the heterogenous compound nucleus :-

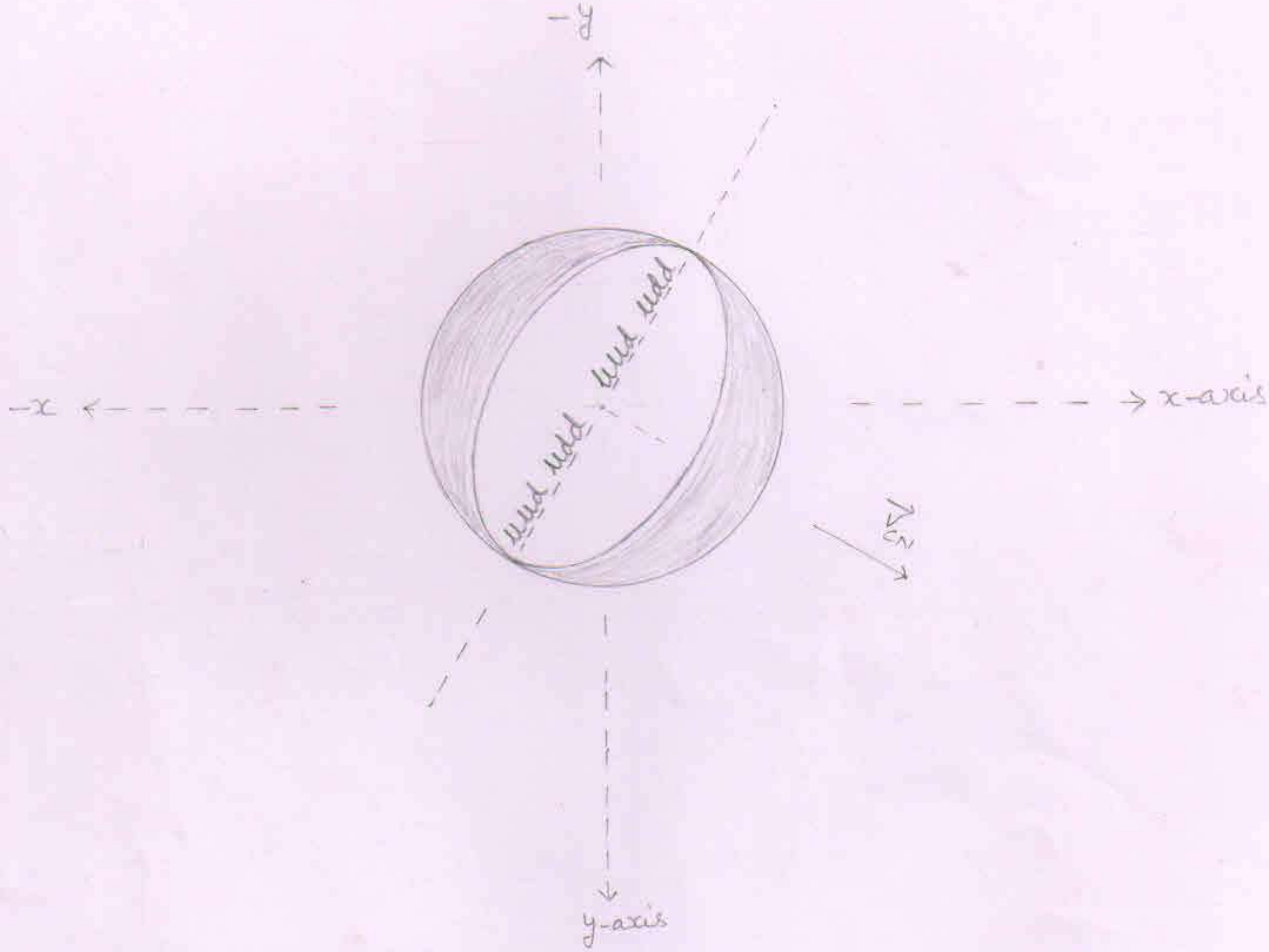
The homogenous compound nucleus [${}^{4}_{2}M$] is unstable. So, for stability, the central group of quarks with its surrounding gluons to become a stable and the just lower nucleus (the helion-4) than the homogenous one [${}^{4}_{2}M$], includes the other 3 groups of quarks with their surrounding gluons and rearrange to form the 'A' lobe of the heterogenous compound nucleus.

While the remaining gluons [the gluons (or the mass) that are not involved in the formation of the lobe 'A'] rearrange to form the 'B' lobe of the heterogenous compound nucleus.

Thus, due to formation of two lobes within into the homogenous compound nucleus, the homogenous compound nucleus transforms into the heterogenous compound nucleus.

* The homogenous compound nucleus [${}^{4}_{2}M$] has more mass than the helium-4 nucleus.

Formation of lobes within into the homogenous Compound nucles :-



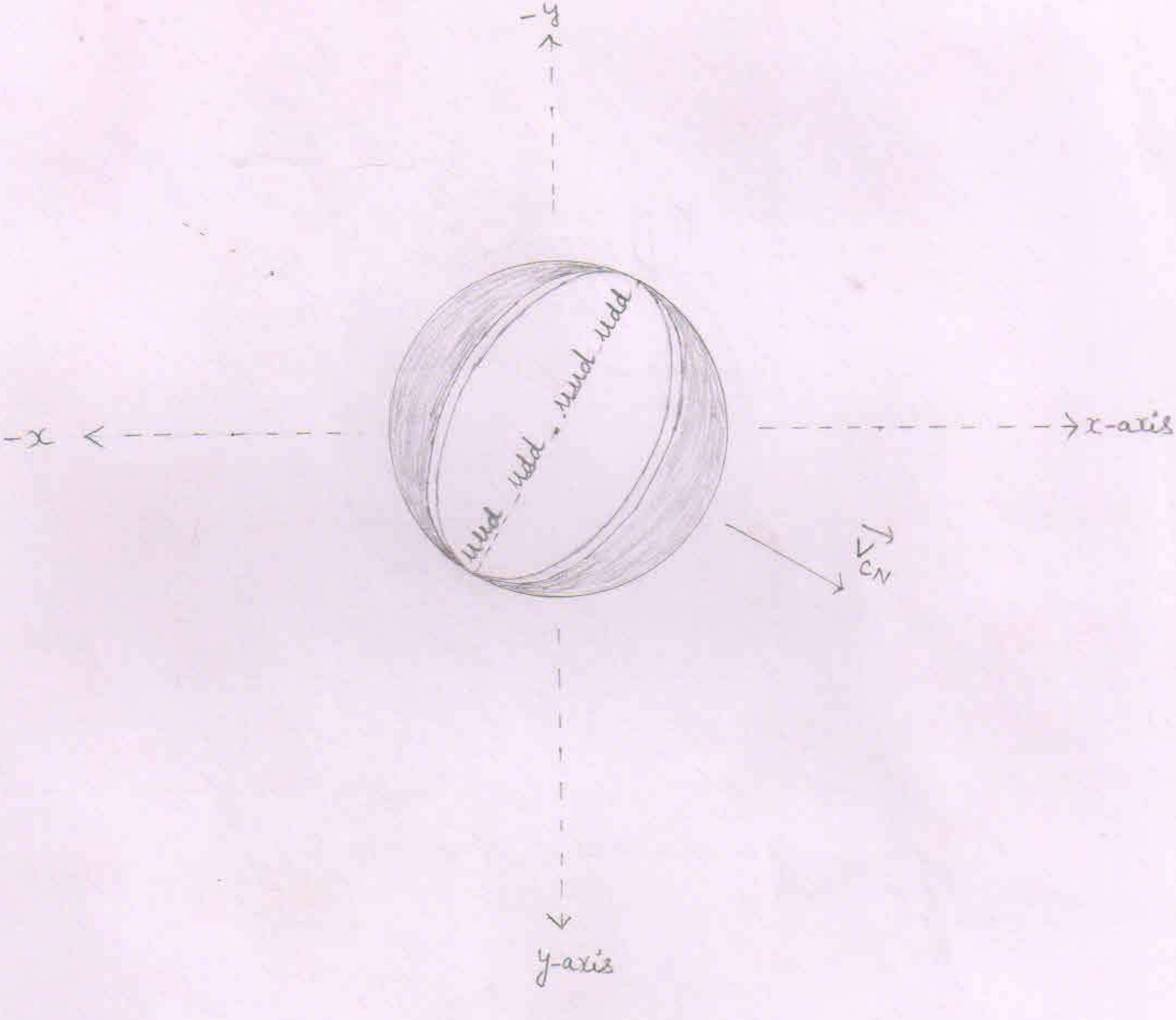
⇒ Where

1. Inner side – lobe 'A' is formed [That is helium-4 nucleus is formed]

2. outer side – The remaining gluons [or the reduced mass]

4. Final stage of the heterogenous compound nucleus:-
The remaining gluons [that compose the 'B' lobe of the heterogenous compound nucleus] remains loosely bonded to the helium-4 nucleus [that compose the 'A' lobe of the heterogenous compound nucleus]. Thus the heterogenous compound nucleus, finally, becomes like a coconut into which the outer shield is made up of the remaining gluons while the inner part is made up of the helium-4 nucleus.

Final stage of the heterogenous compound nucleus :-



The Splitting of the heterogenous compound nucleus

⇒ The remaining gluons are loosely bonded to the helium-4 nucleus.

⇒ At the poles of the helium-4 nucleus, the remaining gluons are lesser in amount than at the equator.

⇒ So, during the rearrangement of the remaining gluons [or during the formation of the 'B' lobe of the heterogenous compound nucleus], the remaining gluons to be homogenously distributed all around, rush from the equator to the poles.
In this way, the loosely bonded remaining gluons separates from the helium-4 nucleus and also divides itself into two parts giving us three particles - the first one is the one-half of the reduced mass, second one is the helium-4 nucleus and the third one is the one-half of the reduced mass

⇒ Thus the heterogenous compound nucleus splits according to the lines parallel to the velocity of the compound nucleus into three particles - the first one is the one-half of the reduced mass $\left(\frac{\Delta m}{2}\right)$, the second one is the helium-4 nucleus and the third one is the another one-half of the reduced mass $\left(\frac{\Delta m}{2}\right)$.

⇒ By the law of inertia, each particle that has separated from the compound nucleus has an inhereted velocity $(\vec{V}_{inh})$ equal to to the velocity of the compound nucleus $(\vec{V}_{CN})$.

⇒ So, for conservation of momentum

$$M\vec{V}_{CN} = \left(\frac{\Delta m}{2} + m_{he\text{-}4} + \frac{\Delta m}{2}\right)\vec{V}_{CN}$$

where,

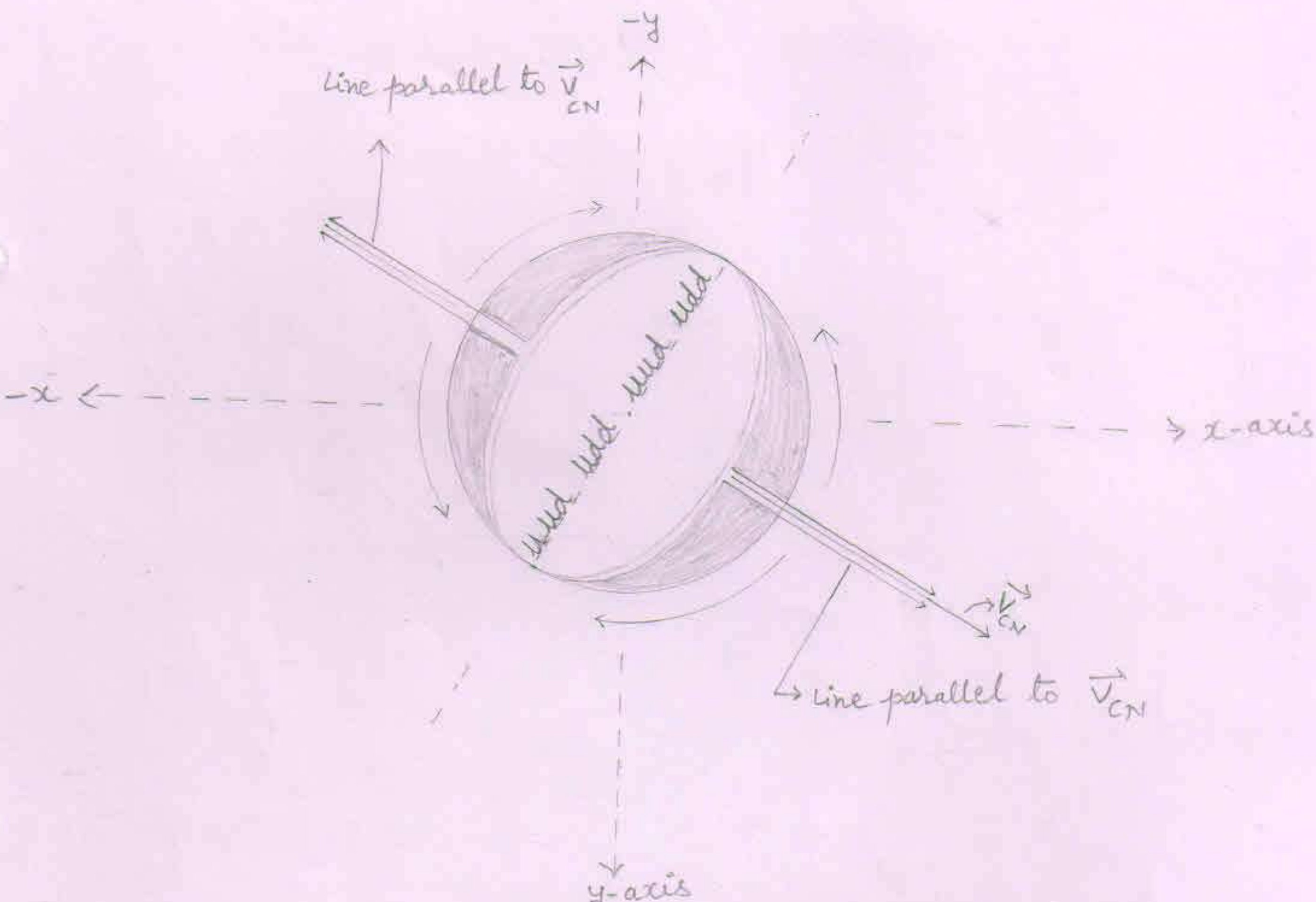
M = Mass of the compound nucleus

$\vec{V}_{CN}$ = velocity of the compound nucleus

$\frac{\Delta m}{2}$ = one-half of the reduced mass

$m_{he\text{-}4}$ = mass of the helium-4 nucleus.

The splitting of the heterogenous compound nucleus :-



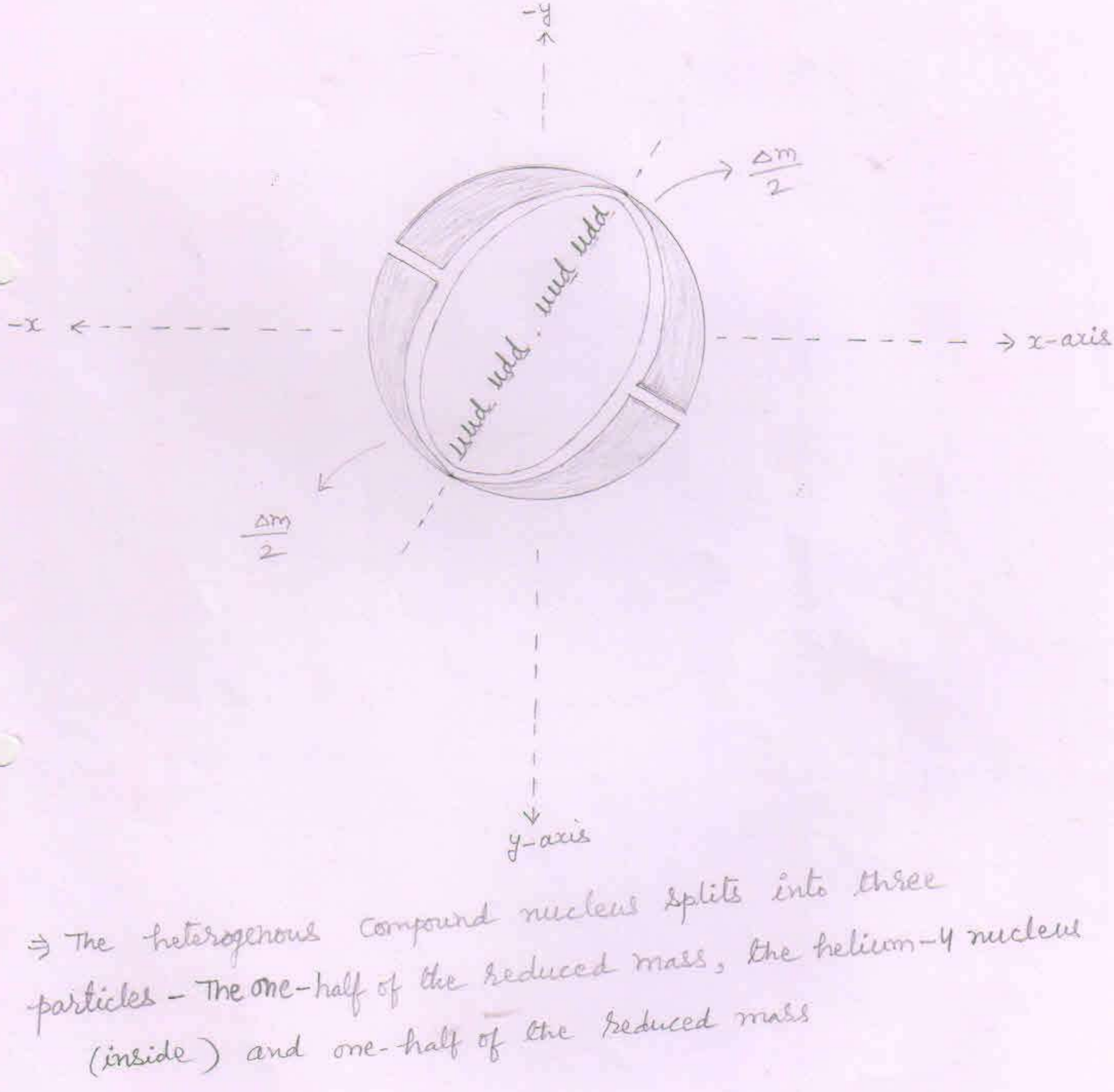
⇒ The remaining gluons are loosely bonded to the helium-4 nucleus.

⇒ At the poles of the helium-4 nucleus, the remaining gluons are lesser in amount than at the equitorial. So, during the rearrangement of the remaining gluons [or during the formation of the 'B' lobe of the heterogenous compound nucleus], the remaining gluons, for balance, rush from the equator to poles.

In this way, the loosely bonded remaining gluons separates from the helium-4 nucleus giving us three separate particles - helium-4 nucleus, $\frac{\Delta m}{2}$ and $\frac{\Delta m}{2}$

Thus, the heterogenous compound nucleus splits according to the lines parallel to the velocity of the compound nucleus into

The splitting of the heterogenous compound nucleus :-



⇒ The heterogenous compound nucleus splits into three particles - The one-half of the reduced mass, the helium-4 nucleus (inside) and one-half of the reduced mass

Inhereted velocity ($\vec{V}_{inh}$) of the particles :-

Each particle that has separated from the compound nucleus has an inhereted velocity ($\vec{V}_{inh}$) equal to the velocity of the compound nucleus ($\vec{V}_{CN}$).

I. Inhereted velocity of the helium-4 nucleus

$$V_{inh} = V_{CN} = 0.3089 \times 10^{7} \text{ m/s}$$

⇒ components of the inhereted velocity of the helium-4 nucleus :-

1. $\vec{V}_x = V_{inh} \cos\alpha = V_{CN} \cos\alpha = 0.2676 \times 10^{7}$ m/s

2. $\vec{V}_y = V_{inh} \cos\beta = V_{CN} \cos\beta = 0.1545 \times 10^{7}$ m/s

3. $\vec{V}_z = V_{inh} \cos\gamma = V_{CN} \cos\gamma = 0$ m/s

II. Inhereted velocity of the each one-half of the reduced mass :-

$$V_{inh} = V_{CN} = 0.3089 \times 10^{7} \text{ m/s}$$

Propellation of the particles

1. Reduced mass

$$\Delta m = [m_d + m_d] - [m_{he-4}]$$

$$\Delta m = [2 \times 2.01355] - [4.0015] \text{ amu}$$

$$\Delta m = [4.0271] - [4.0015] \text{ amu}$$

$$\Delta m = 0.0256 \text{ amu}$$

$$\Delta m = 0.0256 \times 1.6605 \times 10^{-27} \text{ Kg}$$

$$\Delta m = 0.0425088 \times 10^{-27} \text{ Kg}$$

2. Inhereted kinetic energy of the reduced mass

$$E_{inh} = \frac{1}{2} \Delta m \, V_{inh}^2 = \frac{1}{2} \Delta m \, V_{CN}^2$$

$$V_{CN}^2 = 0.09548001 \times 10^{14} \text{ m}^2/\text{s}^2$$

$$\Rightarrow E_{inh} = \frac{1}{2} \times 0.0425088 \times 10^{-27} \times 0.09548001 \times 10^{14} \text{ J}$$

$$\Rightarrow E_{inh} = 0.00202937032 \times 10^{-13} \text{ J}$$

$$\Rightarrow E_{inh} = 0.0012 \text{ Mev}$$

3. $$E_{Released} = \Delta m c^2$$

$$= 0.0256 \times 931 \text{ Mev}$$

$$= 23.8336 \text{ Mev}$$

4. $$E_{Total} = E_{inhereted} + E_{Released}$$

$$= [0.0012] + [23.8336] \text{ Mev}$$

$$= 23.8348 \text{ Mev}$$

Propellation of the particles

⇒ The each one-half of the reduced mass ($\Delta m/2$) converts into energy. So, the energy (E) carried by the produced pairs of gamma ray photons is –

$$E = \frac{E_T}{2}$$

$$E_T = 23.8348 \text{ Mev}$$

$$\Rightarrow E = \frac{23.8348}{2} \text{ Mev}$$

$$\Rightarrow E = 11.9174 \text{ Mev}$$

⇒ When a pair of gamma ray photon make a head-on collision with a nucleus, it imparts its extra energy to the nucleus by a pair production [that is producing a positron and an electron].

So, for pair production each pair of gamma ~~p~~ ray photon carrying high energy must have an energy (1.02 Mev) equal to or greater than the sum of the energies – the energy equal to the rest mass of the positron ($m_{e^+} c^2$) and the energy equal to the rest mass of the an electron ($m_{e^-} c^2$).

Number of pairs of gamma ray photons (N_γ) :-

⇒ When one-half of the reduced mass ($\Delta m/2$) converts into energy, the energy (E) carried by the pairs of gamma ray photons is 11.9174 Mev.

⇒ Each pair of gamma ray photon that carry a part of the energy (E) must have an energy equal to or more than 1.02 Mev.

⇒ So,

$$\left[\begin{array}{l}\text{Number of} \\ \text{pairs of gamma} \\ \text{ray photons}\end{array}\right] = \left[\begin{array}{l}\dfrac{\text{Energy (E) produced due to } \Delta m/2}{\text{Energy that must carried by a pair of g.r. photon}}\end{array}\right]$$

$$\Rightarrow \quad N_\gamma = \frac{11.9174}{1.02} \frac{\text{Mev}}{\text{Mev}}$$

$$\Rightarrow \quad N_\gamma = 11.68$$

⇒ Taking the whole digit, we may say that there are 11 pairs of gamma ray photons that carry the energy 11.9174 Mev.

⇒ Thus there are the 22 pairs of gamma ray photons that carry the total energy (E_T) equal to 23.8348 Mev.

Page No.:
Date : / /

Energy carried by the each pair of gamma ray photon [E_γ]:

$\Rightarrow$ Energy carried by the each pair of gamma ray photon is equal to the energy (E) produced due to the one-half of the reduced mass ($\Delta m/2$) divided by the total number of pairs of gamma ray photons that carry the energy (E).

$$\Rightarrow E_\gamma = \frac{\text{Energy (E) produced due to } \Delta m/2}{\text{Total number of pairs of g.r. photons that carry energy (E)}}$$

$$\Rightarrow E_\gamma = \frac{11.9174}{11} \text{ Mev}$$

$$\Rightarrow E_\gamma = 1.0834 \text{ Mev}$$

Conclusion : Each pair of gamma ray photon carry 1.0834 Mev energy.

Propellation of the particle :-

As the reduced mass converts into energy, the total energy (E_T) is carried away by the gamma ray photons.

⇒ Conservation of momentum :

We know that the reduced mass has separated from the compound nucleus with an inhereted velocity ($\vec{V}_{inh}$) equal to the velocity of the compound nucleus ($\vec{V}_{CN}$).

So, for conservation of momentum, if we add up the momenta of all the produced gamma ray photons, we will again get the total momentum equal to the reduced mass multiplied by the velocity of the compound nucleus ($\Delta m \vec{V}_{CN}$).

Now, if we denote the momentum of a pair of gamma ray photon by $\vec{P}_\gamma$ then,

The total momentum of all the produced 22 pairs of gamma ray photons will be equal to $\Delta m \vec{V}_{CN}$.

or

$$(\vec{P}_{\gamma(1)} + \vec{P}_{\gamma(2)} + \vec{P}_{\gamma(3)} + \vec{P}_{\gamma(4)} + \vec{P}_{\gamma(5)} + \vec{P}_{\gamma(6)} + \vec{P}_{\gamma(7)} + \vec{P}_{\gamma(8)}$$

$$+ \vec{P}_{\gamma(9)} + \vec{P}_{\gamma(10)} + \vec{P}_{\gamma(11)} + \vec{P}_{\gamma(12)} + \vec{P}_{\gamma(13)} + \vec{P}_{\gamma(14)} + \vec{P}_{\gamma(15)}$$

$$+ \vec{P}_{\gamma(16)} + \vec{P}_{\gamma(17)} + \vec{P}_{\gamma(18)} + \vec{P}_{\gamma(19)} + \vec{P}_{\gamma(20)} + \vec{P}_{\gamma(21)}$$

$$+ \vec{P}_{\gamma(22)} = \Delta m \vec{V}_{CN}$$

2. We know that the inhereted kinetic energy of the reduced mass is negligible. So, we can take the inhereted momentum of the reduced mass ($\Delta m \vec{V}_{CN}$) is equal to zero.

If we take that the inhereted momentum of the reduced mass ($\Delta m \vec{V}_{CN}$) is equal to zero. Then the sum of the momenta of all the produced 22 pairs of gamma ray photons will be equal to zero.

i.e.

$$\vec{P}_{\gamma(1)} + \vec{P}_{\gamma(2)} + \vec{P}_{\gamma(3)} + \vec{P}_{\gamma(4)} + \vec{P}_{\gamma(5)} + \vec{P}_{\gamma(6)} + \vec{P}_{\gamma(7)} + \vec{P}_{\gamma(8)}$$

$$+ \vec{P}_{\gamma(9)} + \vec{P}_{\gamma(10)} + \vec{P}_{\gamma(11)} + \vec{P}_{\gamma(12)} + \vec{P}_{\gamma(13)} + \vec{P}_{\gamma(14)} + \vec{P}_{\gamma(15)}$$

$$+ \vec{P}_{\gamma(16)} + \vec{P}_{\gamma(17)} + \vec{P}_{\gamma(18)} + \vec{P}_{\gamma(19)} + \vec{P}_{\gamma(20)} + \vec{P}_{\gamma(21)} + \vec{P}_{\gamma(22)} = 0$$

So, we reach at this conclusion that out of 22 pairs of gamma ray photons, the sum of momenta of 11 pairs of gamma ray photons is equal and opposite to the sum of momenta of rest 11 pairs of gamma ray photons.

Or we may say that there is a pair of gamma ray photon having equal and opposite momentum to the another pair of gamma ray photon.

3. When one-half of the reduced mass ($\Delta m/2$) converts into energy, the one-half of the total energy ($\frac{E_T}{2}$) is carried away by the 11 pairs of gamma ray photons.

The pair of gamma ray photon numbered as '2' travelling in I quadrant has equal and opposite momentum to the pair of gamma ray photon numbered as '13' travelling in the III quadrant.

Similarly, the pair of gamma ray photon numbered as '6' travelling in the II quadrant has equal and opposite momentum to the pair of gamma ray photon numbered as '17' travelling in the IV quadrant.

That is,

$$\vec{P}_{\gamma(2)} = -\vec{P}_{\gamma(13)}$$

Similarly

$$\vec{P}_{\gamma(6)} = -\vec{P}_{\gamma(17)}$$

4. The sum of the momenta of the pairs of gamma ray photons travelling in the I quadrant is equal and opposite to the sum of the momenta of the pairs of gamma ray photons travelling in the III quadrant.

Similarly, the sum of the momenta of the pairs of gamma ray photons travelling in the II quadrant is equal and opposite to the sum of the momenta of the pairs of gamma ray photons travelling in the IV quadrant.

Conclusion : all the produced 22 pairs of gamma ray photons travel towards the helium-4 nucleus so that the momentum is conserved.

In this condition, each pair of gamma ray photons strike or make a head-on collision to the helium-4 nucleus and imparts its extra energy to the helium-4 nucleus.

Page No.:
Date : / /

Increased energy (E_{inc}) of the helium-4 nucleus :-

⇒ Each pair of gamma ray photon carry 1.0834 MeV energy.

⇒ Each pair of gamma ray photon by making a head-on collision with helium-4 nucleus imparts its extra energy to the helium-4 nucleus.

⇒ Extra energy of a pair of gamma ray photon is equal to the energy (E_γ) carried by a pair of gamma ray photon minus 1.02 MeV.

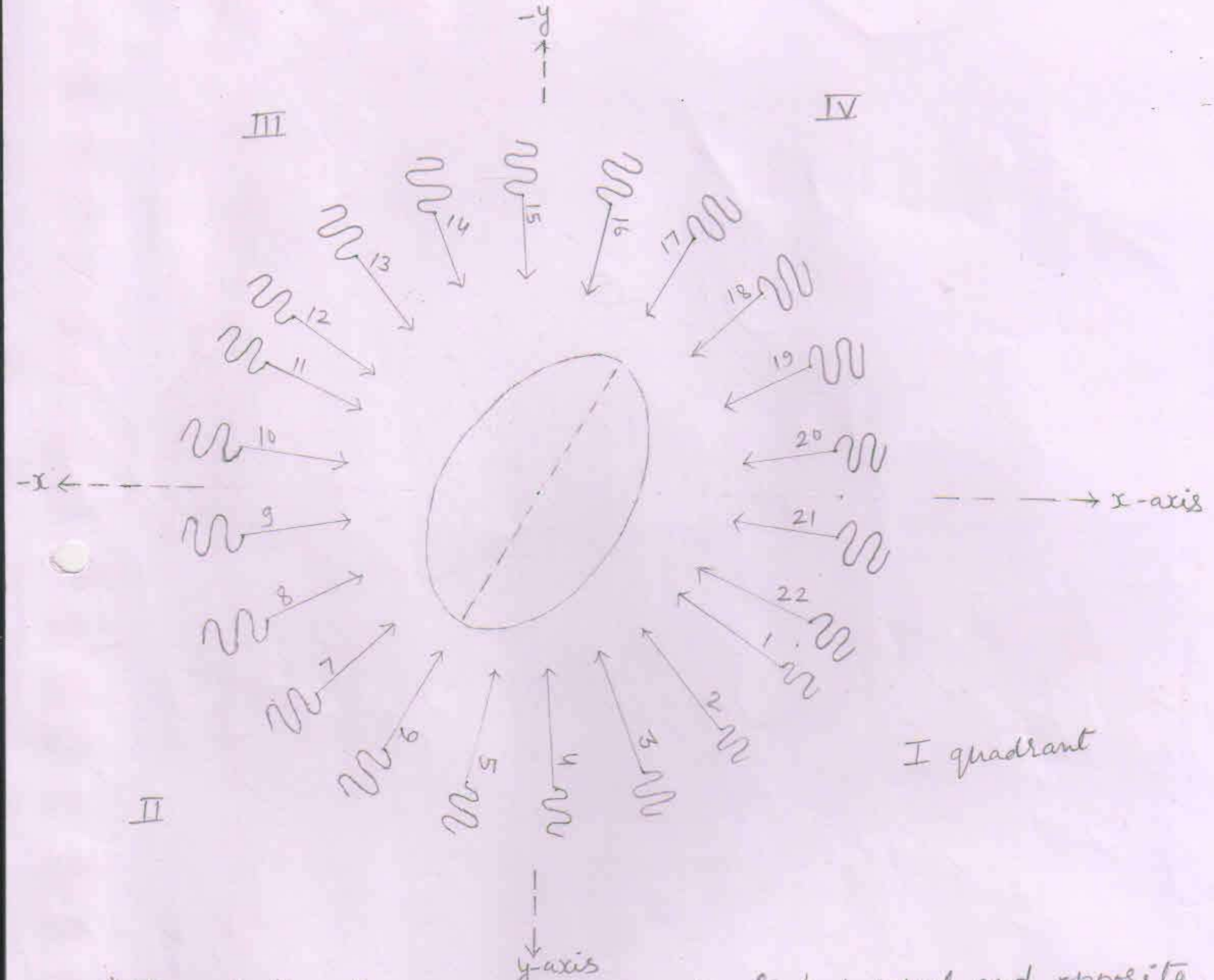
$$E_{extra} = E_\gamma - [m_{e^+}c^2 + m_{e^-}c^2]$$

$$= 1.0834 - [1.02] \text{ MeV}$$
$$= 0.0634 \text{ MeV}$$

⇒ So, when the 22 pairs of gamma ray photons strike to the helium-4 nucleus, the increased energy (E_{inc}) of the helium-4 nucleus is -

$$E_{inc} = E_{\text{imparted by a pair of g.r. photon}} \times 22$$

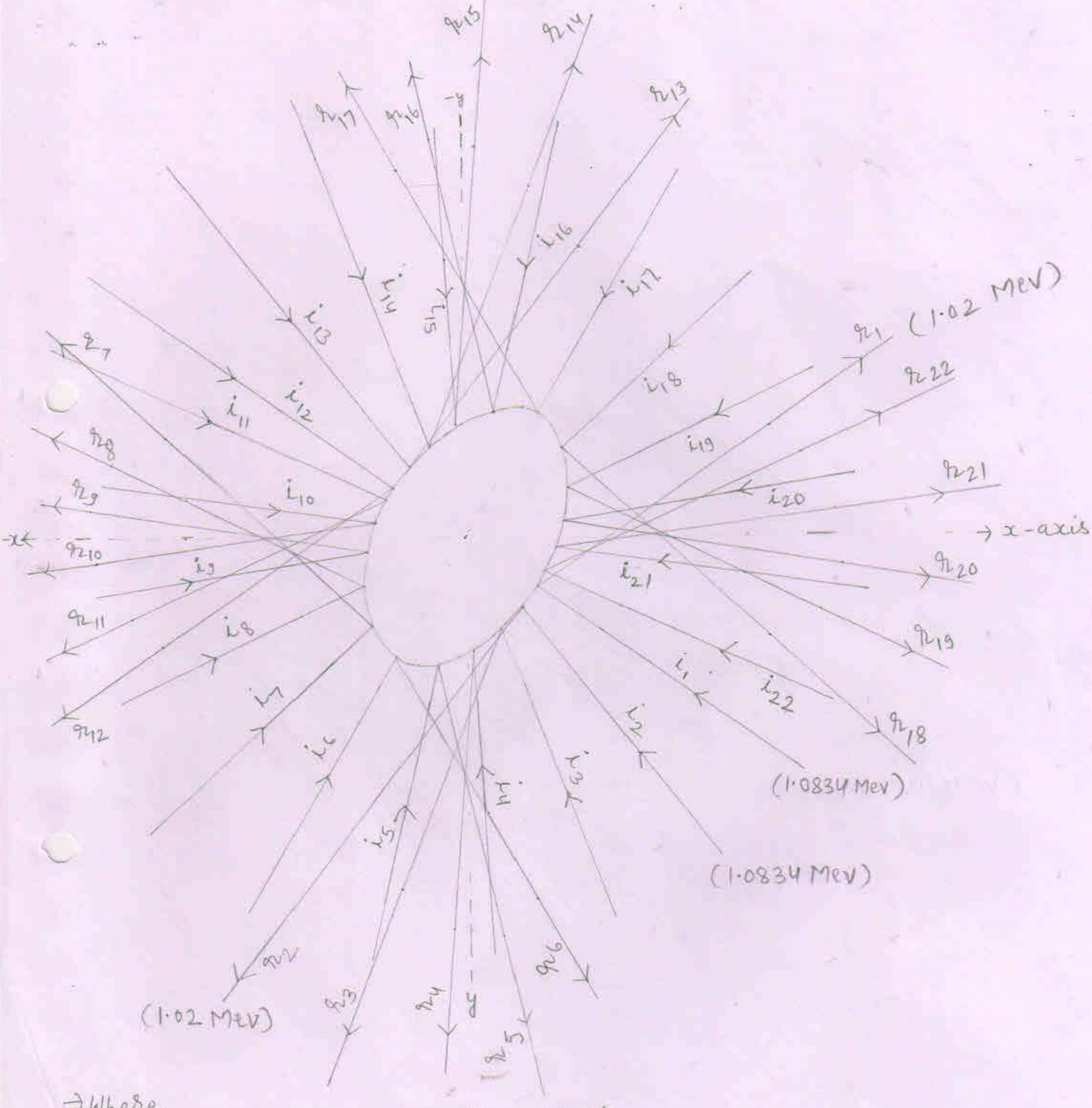
$$\Rightarrow E_{inc} = 0.0634 \text{ MeV} \times 22$$
$$= 1.3948 \text{ MeV}.$$



⇒ The photon travelling in IV quadrant has equal and opposite momentum to the photon travelling in the II quadrant.
The photon that has numbered as '17' has equal and opposite momentum to the photon that is numbered as '6'

⇒ Similarly, the pair of photon that is numbered as '22' travelling in the I quadrant has equal and opposite momentum to the pair of photon that is numbered as '11' and travelling in the III quadrant.

⇒ Each pair of gamma ray photon carry 1.0834 Mev energy



⇒ Where,

i_1 = incident photon first

r_1 = reflected photon first

⇒ Each incident pair of gamma ray photon carry 1.0834 Mev energy and each reflected pair of gamma ray photon carry 1.02 Mev energy.

⇒ The angle of incidence is equal to the angle of reflection

⇒ The helium-4 nucleus is energised by the 1.3948 Mev energy.

Page No.:
Date : / /

Increased velocity (v_{inc}) of the helium-4 nucleus

$$v_{inc} = \left[\frac{2 E_{inc}}{m_{he-4}} \right]^{\frac{1}{2}}$$

$$E_{inc} = 1.3948 \text{ Mev}$$

$$\Rightarrow v_{inc} =$$

$$\Rightarrow v_{inc} = \left[\frac{2 \times 1.3948 \times 1.6 \times 10^{-13}}{6.64449 \times 10^{-27}} \right]^{\frac{1}{2}} \text{ m/s}$$

$$\Rightarrow v_{inc} = \left[\frac{4.46336}{6.64449} \times 10^{14} \right]^{\frac{1}{2}} \text{ m/s}$$

$$\Rightarrow v_{inc} = \left[0.6717385382 4 \times 10^{14} \right]^{\frac{1}{2}} \text{ m/s}$$

$$\Rightarrow \bar{v}_{inc} = 0.8195 \times 10^{7} \text{ m/s}$$

Components of the increased velocity of the helium-4 nucleus:-

⇒ We know that the helium-4 nucleus has separated from the compound nucleus with an inhereted velocity ($\vec{V}_{inh}$) which is equal to the velocity of the compound nucleus ($\vec{V}_{CN}$).

⇒ The inhereted velocity (as well as the velocity of compound nucleus) of the helium-4 nucleus makes angle 30° with x-axis, 60° angle with y-axis and 90° angle with z-axis.

⇒ So, we may say that the helium-4 nucleus moving in a direction that make angle 30° with x-axis, 60° angle with y-axis and 90° angle with z-axis is energised by the 1.3948 MeV energy due to head-on collision between the gamma ray photon(s) and the helium-4 nucleus.

⇒ So, the increased velocity ($\vec{V}_{inc}$) of the helium-4 nucleus also make angle 30° with x-axis, 60° angle with y-axis and the 90° angle with z-axis. So,

1. $\vec{V}_x = V_{inc} \cos\alpha$

$V_{inc} = 0.8195 \times 10^7$ m/s

$\cos\alpha = \cos 30° = 0.866$

$\Rightarrow \vec{V}_x = 0.8195 \times 10^7 \times 0.866$ m/s

$= 0.7096 \times 10^7$ m/s

2. $\vec{V}_y = V_{inc} \cos\beta$

$\cos\beta = \cos 60° = 0.5$

$\Rightarrow \vec{V}_y = 0.8195 \times 10^7 \times 0.5$ m/s

$= 0.4097 \times 10^7$ m/s

3. $\vec{V}_z = V_{inc} \cos\gamma$

$\cos\gamma = \cos 90° = 0$

$\Rightarrow \vec{V}_z = 0.8195 \times 10^7 \times 0$ m/s $= 0$ m/s

Page No.:
Date : / /

components of the final velocity ($\vec{v}_f$) of helium-4 nucleus

According to	Inhereted velocity ($\vec{v}_{inh}$)	Increased velocity ($\vec{v}_{inc}$)	Final velocity ($\vec{v}_f$) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = 0.2676 \times 10^7$ m/s	$\vec{v}_x = 0.7096 \times 10^7$ m/s	$\vec{v}_x = 0.9772 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.1545 \times 10^7$ m/s	$\vec{v}_y = 0.4097 \times 10^7$ m/s	$\vec{v}_y = 0.5642 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

Page No.:
Date : / /

Final velocity (v_f) of the helium-4 nucleus :-

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$v_x = 0.9772 \times 10^7$ m/s
$v_y = 0.5642 \times 10^7$ m/s
$v_z = 0$ m/s

$$\Rightarrow v_f^2 = (0.9772 \times 10^7)^2 + (0.5642 \times 10^7)^2 + (0)^2 \quad m^2/s^2$$

$$\Rightarrow v_f^2 = (0.95491984 \times 10^{14}) + (0.31832164 \times 10^{14}) + (0) \quad m^2/s^2$$

$$\Rightarrow v_f^2 = 1.27324148 \times 10^{14} \quad m^2/s^2$$

$$\Rightarrow v_f = 1.1283 \times 10^7 \quad m/s$$

Final kinetic energy of the helium-4 nucleus

$$E = \frac{1}{2} m_{he\text{-}4} v_f^2$$

$v_f^2 = 1.27324148 \times 10^{14}$ m²/s²

$$\Rightarrow E = \frac{1}{2} \times 6.64449 \times 10^{-27} \times 1.27324148 \times 10^{14} \quad J$$

$$= 4.23002014072 \times 10^{-13} \quad J$$

$$= 2.6437 \quad MeV$$

$$\Rightarrow m_{he\text{-}4} v_f^2 = 6.64449 \times 10^{-27} \times 1.27324148 \times 10^{14} \quad J$$

$$= 8.4600 \times 10^{-13} \quad J$$

Forces acting on the helium-4 nucleus

1. $F_y = q V_x B_z \sin\theta$

$\vec{V}_x = 0.9772 \times 10^{7}$ m/s
$\vec{B}_z = -1$ Tesla
$q = 2 \times 1.6 \times 10^{-19}$ C
$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_y = 2 \times 1.6 \times 10^{-19} \times 0.9772 \times 10^{7} \times 1 \times 1$ N
$= 3.1270 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_y$ is according to negative y-axis. So,
$\vec{F}_y = -3.1270 \times 10^{-12}$ N

2. $F_z = q V_x B_y \sin\theta$

$\vec{B}_y = 1$ Tesla
$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_z = 2 \times 1.6 \times 10^{-19} \times 0.9772 \times 10^{7} \times 1 \times 1$ N
$= 3.1270 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_z$ is according to negative z-axis. So,
$\vec{F}_z = -3.1270 \times 10^{-12}$ N

3. $F_x = q V_y B_z \sin\theta$

$\vec{V}_y = 0.5642 \times 10^{7}$ m/s
$\vec{B}_z = -1$ Tesla
$\sin\theta = \sin 90^\circ = 1$

$\Rightarrow F_x = 2 \times 1.6 \times 10^{-19} \times 0.5642 \times 10^{7} \times 1 \times 1$ N
$= 1.8054 \times 10^{-12}$ N

From the right hand palm rule, the direction of force $\vec{F}_x$ is according to positive x-axis. So,
$\vec{F}_x = 1.8054 \times 10^{-12}$ N

4. Resultant force (F_R) acting on the helium-4 nucleus :-

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$F_x = 1.8054 \times 10^{-12}$ N

$F = F_y = F_z = 3.1270 \times 10^{-12}$ N

$$\Rightarrow F_R^2 = F_x^2 + 2F^2$$

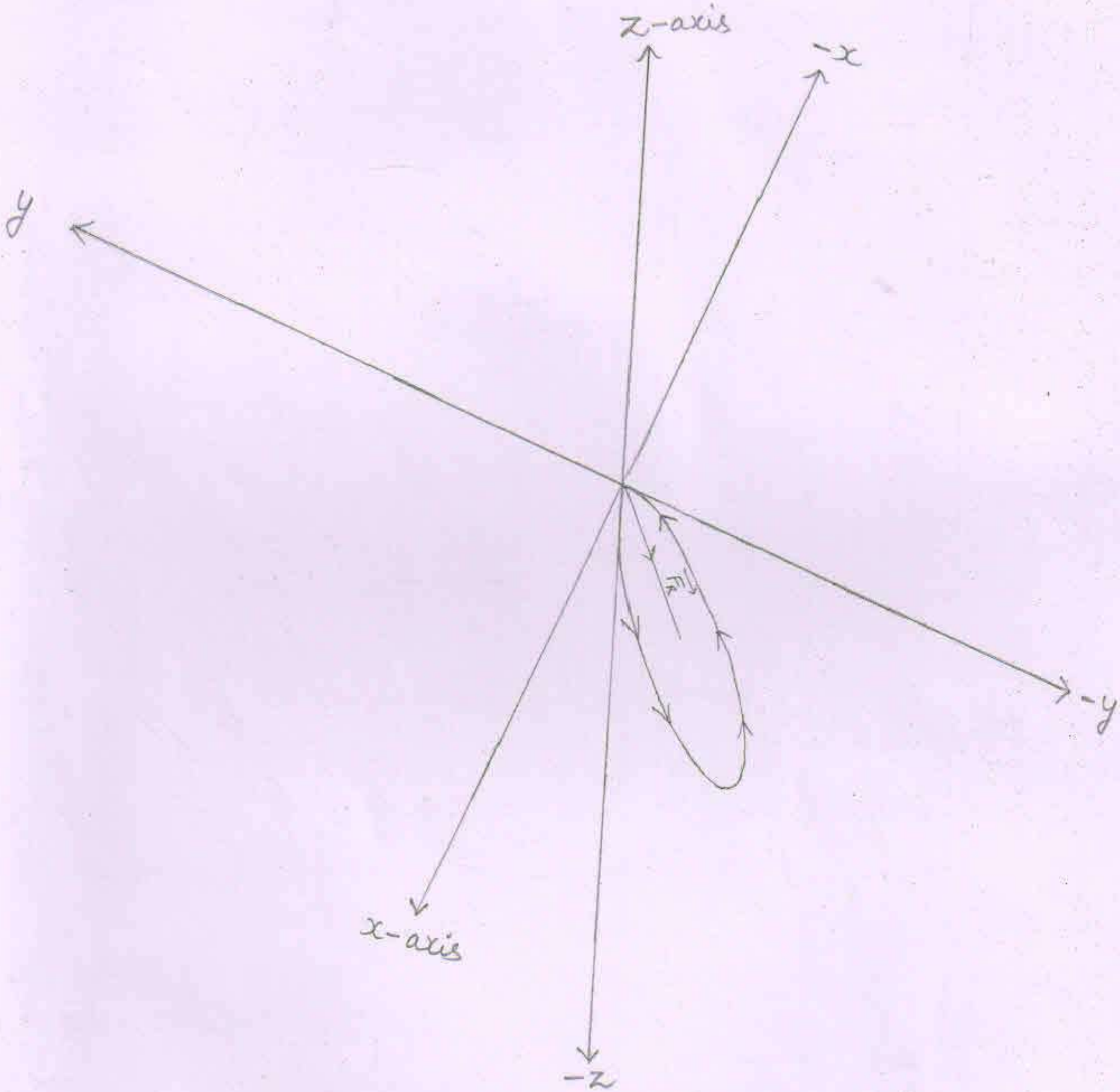
$$\Rightarrow F_R^2 = (1.8054 \times 10^{-12})^2 + 2(3.1270 \times 10^{-12})^2 \ N^2$$

$$\Rightarrow F_R^2 = (3.25946916 \times 10^{-24}) + 2(9.778129 \times 10^{-24}) \ N^2$$

$$\Rightarrow F_R^2 = (3.25946916 \times 10^{-24}) + (19.556258 \times 10^{-24}) \ N^2$$

$$\Rightarrow F_R^2 = 22.81572716 \times 10^{-24} \ N^2$$

$$\Rightarrow F_R = 4.7765 \times 10^{-12} \ N$$



⇒ The circular orbit to be followed by the helion-4 lies in the plane made up of positive x-axis, negative y-axis and the negative z-axis.

⇒ $C_{he\text{-}4}$ = center of the circular orbit to be followed by the helium-4 nucleus.

⇒ $\overrightarrow{F_R}$ = Resultant force

Page No.:
Date : / /

Angles that make the resultant force ($\vec{F}_R$) [acting on the helium-4 nucleus when the helium-4 nucleus is at point 'F'] with positive x, y, and z axes.

1. With x-axis

$$\cos\alpha = \frac{F_R \cos\alpha}{F_R} = \frac{\vec{F_x}}{F_R} = \frac{1.8054 \times 10^{-12}}{4.7765 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\alpha = 0.3779$

$\Rightarrow \alpha \simeq 67.8$ degree $[\because \cos(67.8) = 0.3778]$

2. With y-axis

$$\cos\beta = \frac{F_R \cos\beta}{F_R} = \frac{\vec{F_y}}{F_R} = \frac{-3.1270 \times 10^{-12}}{4.7765 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\beta = -0.6546$

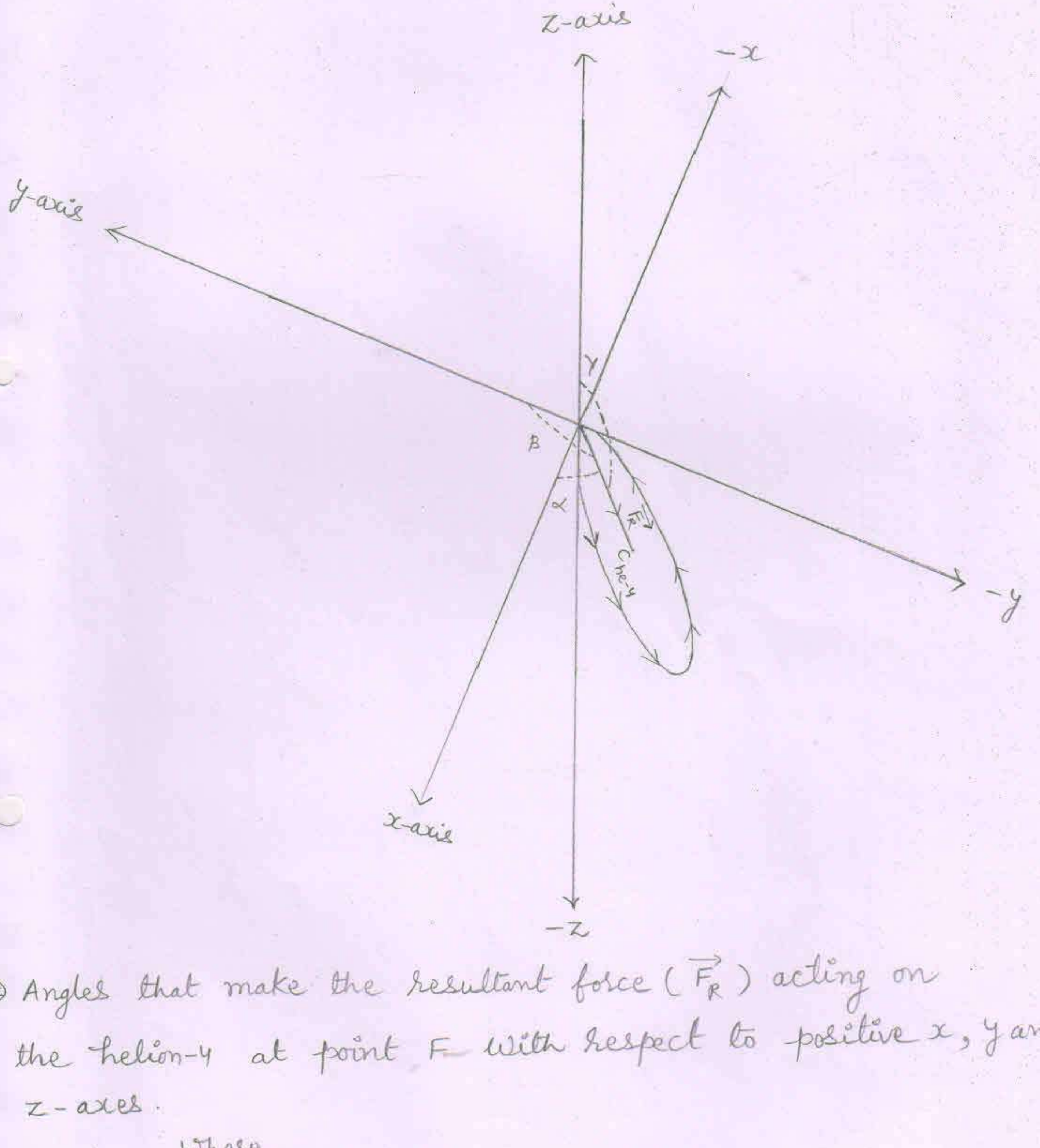
$\Rightarrow \beta \simeq 130.8$ degree $[\because \cos(130.8) = -0.6534]$

3. With z-axis

$$\cos\gamma = \frac{F_R \cos\gamma}{F_R} = \frac{\vec{F_z}}{F_R} = \frac{-3.1270 \times 10^{-12}}{4.7765 \times 10^{-12}} \frac{N}{N}$$

$\Rightarrow \cos\gamma = -0.6546$

$\Rightarrow \gamma \simeq 130.8$ degree



⇒ Angles that make the resultant force ($\vec{F_R}$) acting on the helion-4 at point F with respect to positive x, y and z - axes.

Where,

$\alpha \simeq 67.8$ degree

$\beta \simeq 130.8$ degree

$\gamma = 130.8$ degree

5. Radius of the circular orbit followed by the helium-4 nucleus :-

$$r = \frac{mv^2}{F_R}$$

$$mv^2 = 8.4600 \times 10^{-13} \text{ J}$$
$$F_R = 4.7765 \times 10^{-12} \text{ N}$$

$$\Rightarrow r = \frac{8.4600 \times 10^{-13}}{4.7765 \times 10^{-12}} \frac{\text{J}}{\text{N}}$$

$$\Rightarrow r = 1.77117 \times 10^{-1} \text{ m}$$

$$\Rightarrow r = 17.7117 \times 10^{-2} \text{ m}$$

The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the helium-4 nucleus.

1. $\cos\alpha = \frac{x_2 - x_1}{d}$

$$d = 2 \times r$$
$$= 2 \times 17.7117 \times 10^{-2}\ m$$
$$= 35.4234 \times 10^{-2}\ m$$
$$\cos\alpha = 0.37$$

$\Rightarrow x_2 - x_1 = d \times \cos\alpha$

$\Rightarrow x_2 - x_1 = 35.4234 \times 10^{-2} \times 0.37\ m$

$\Rightarrow x_2 - x_1 = 13.1066 \times 10^{-2}\ m$

$\Rightarrow x_2 = 13.1066 \times 10^{-2}\ m \qquad [\because x_1 = 0]$

2. $\cos\beta = \frac{y_2 - y_1}{d}$

$$\cos\beta = -0.65$$

$\Rightarrow y_2 - y_1 = d \times \cos\beta$

$\Rightarrow y_2 - y_1 = 35.4234 \times 10^{-2} \times (-0.65)\ m$

$\Rightarrow y_2 - y_1 = -23.0252 \times 10^{-2}\ m$

$\Rightarrow y_2 = -23.0252 \times 10^{-2}\ m \qquad [\because y_1 = 0]$

3. $\cos\gamma = \frac{z_2 - z_1}{d}$

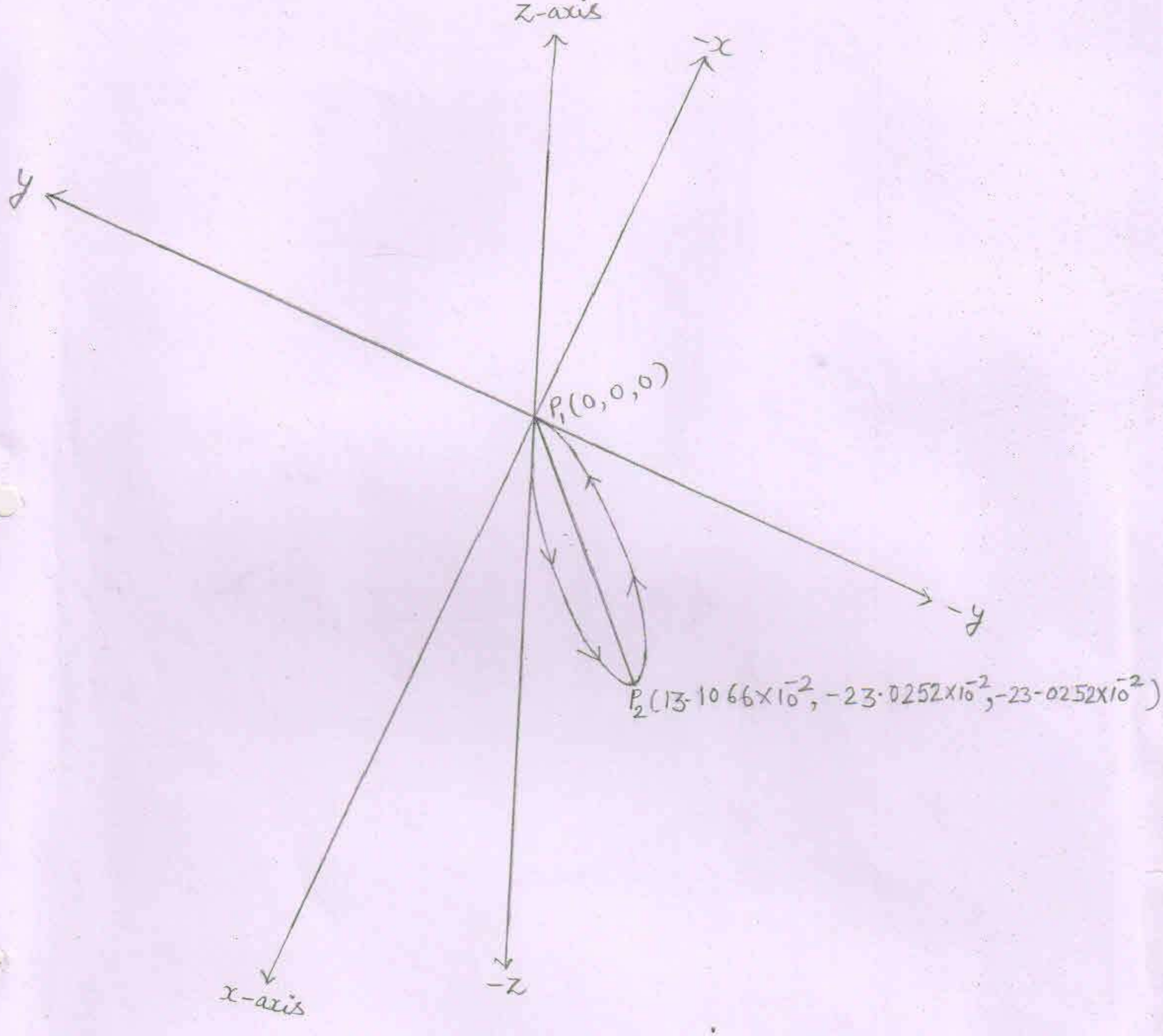
$$\cos\gamma = -0.65$$

$\Rightarrow z_2 - z_1 = d \times \cos\gamma$

$\Rightarrow z_2 - z_1 = 35.4234 \times 10^{-2} \times (-0.65)\ m$

$\Rightarrow z_2 - z_1 = -23.0252 \times 10^{-2}\ m$

$\Rightarrow z_2 = -23.0252 \times 10^{-2}\ m \qquad [\because z_1 = 0]$



⇒ The cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and the $P_2(x_2, y_2, z_2)$ located on the circumfrence of the circle obtained by the helion-4 are as above shown.

⇒ The line $\overline{P_1P_2}$ is the diameter of the circle.