

Components of the increased velocity (\vec{v}_{inc}) of the particles

I. For neutron

$$1. \vec{v}_x = v_{inc} \cos \alpha$$

$$v_{inc} = 1.1923 \times 10^7 \text{ m/s}$$
$$\cos \alpha = \cos(53.8) = 0.59$$

$$\Rightarrow \vec{v}_x = 1.1923 \times 10^7 \times 0.59 \text{ m/s}$$
$$= 0.7034 \times 10^7 \text{ m/s}$$

$$2. \vec{v}_y = v_{inc} \cos \beta$$

$$\cos \beta = \cos(36.2) = 0.80$$

$$\Rightarrow \vec{v}_y = 1.1923 \times 10^7 \times 0.80 \text{ m/s}$$
$$= 0.9538 \times 10^7 \text{ m/s}$$

$$3. \vec{v}_z = v_{inc} \cos \gamma$$

$$\cos \gamma = \cos 90^\circ = 0$$

$$\Rightarrow \vec{v}_z = 1.1923 \times 10^7 \times 0 \text{ m/s}$$
$$= 0 \text{ m/s}$$

II. For helium-4

$$1. \vec{v}_x = v_{inc} \cos \alpha$$

$$v_{inc} = 0.3005 \times 10^7 \text{ m/s}$$

$$\cos \alpha = \cos(233.8) = -\cos(53.8) = -0.59$$

$$\Rightarrow \vec{v}_x = 0.3005 \times 10^7 \times (-0.59) \text{ m/s}$$
$$= -0.1772 \times 10^7 \text{ m/s}$$

$$2. \vec{v}_y = v_{inc} \cos \beta$$

$$\cos \beta = \cos(143.8) = -\cos(36.2) = -0.80$$

$$\Rightarrow \vec{v}_y = 0.3005 \times 10^7 \times (-0.80) \text{ m/s}$$
$$= -0.2404 \times 10^7 \text{ m/s}$$

$$3. \vec{v}_z = v_{inc} \cos \gamma$$

$$\cos \gamma = \cos 90^\circ = 0$$

$$\Rightarrow \vec{v}_z = 0.3005 \times 10^7 \times 0 \text{ m/s} = 0 \text{ m/s}$$

Components of the final velocity (\vec{v}_f) of the particles

I. For neutron

According to	Inherited velocity (\vec{v}_{inh})	Increased velocity (\vec{v}_{inc})	Final velocity (\vec{v}_f) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = -0.956 \times 10^7$ m/s	$\vec{v}_x = 0.7034 \times 10^7$ m/s	$\vec{v}_x = -0.2526 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.6991 \times 10^7$ m/s	$\vec{v}_y = 0.9538 \times 10^7$ m/s	$\vec{v}_y = 1.6529 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

II. For helium-4

According to	Inherited velocity (\vec{v}_{inh})	Increased velocity (\vec{v}_{inc})	Final velocity (\vec{v}_f) $\vec{v}_f = \vec{v}_{inh} + \vec{v}_{inc}$
x-axis	$\vec{v}_x = -0.956 \times 10^7$ m/s	$\vec{v}_x = -0.1772 \times 10^7$ m/s	$\vec{v}_x = -1.1332 \times 10^7$ m/s
y-axis	$\vec{v}_y = 0.6991 \times 10^7$ m/s	$\vec{v}_y = -0.2404 \times 10^7$ m/s	$\vec{v}_y = 0.4587 \times 10^7$ m/s
z-axis	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s	$\vec{v}_z = 0$ m/s

Final velocity (v_f) of the neutron

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_x = 0.2526 \times 10^7 \text{ m/s}$$

$$v_y = 1.6529 \times 10^7 \text{ m/s}$$

$$v_z = 0 \text{ m/s}$$

$$\Rightarrow v_f^2 = (0.2526 \times 10^7)^2 + (1.6529 \times 10^7)^2 + (0)^2 \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f^2 = (0.06380676 \times 10^{14}) + (2.73207841 \times 10^{14}) + (0) \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f^2 = 2.79588517 \times 10^{14} \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f = 1.6720 \times 10^7 \text{ m/s}$$

Final kinetic energy of the neutron

$$E = \frac{1}{2} m_n v_f^2$$

$$v_f^2 = 2.79588517 \times 10^{14} \text{ m}^2/\text{s}^2$$

$$\Rightarrow E = \frac{1}{2} \times 1.6749 \times 10^{-27} \times 2.79588517 \times 10^{14} \text{ J}$$

$$\Rightarrow E = 2.34141403561 \times 10^{-13} \text{ J}$$

$$\Rightarrow E = 1.4633 \text{ MeV}$$

Angles that make the final velocity (\vec{v}_f) of the neutron with axes at point F.

1. With x-axis

$$\cos \alpha = \frac{\vec{v}_x}{v_f}$$

$$\vec{v}_x = -0.2526 \times 10^7 \text{ m/s}$$

$$v_f = 1.6720 \times 10^7 \text{ m/s}$$

$$\Rightarrow \cos \alpha = \frac{-0.2526 \times 10^7 \text{ m/s}}{1.6720 \times 10^7 \text{ m/s}} = -0.1510$$

$$\Rightarrow \alpha \approx 98.7 \text{ degree} \quad [\because \cos(98.7) = -0.1512]$$

2. With y-axis

$$\cos \beta = \frac{\vec{v}_y}{v_f}$$

$$\vec{v}_y = 1.6529 \times 10^7 \text{ m/s}$$

$$\Rightarrow \cos \beta = \frac{1.6529 \times 10^7 \text{ m/s}}{1.6720 \times 10^7 \text{ m/s}} = 0.9885$$

$$\Rightarrow \beta \approx 8.7 \text{ degree} \quad [\cos(8.7) = 0.9884]$$

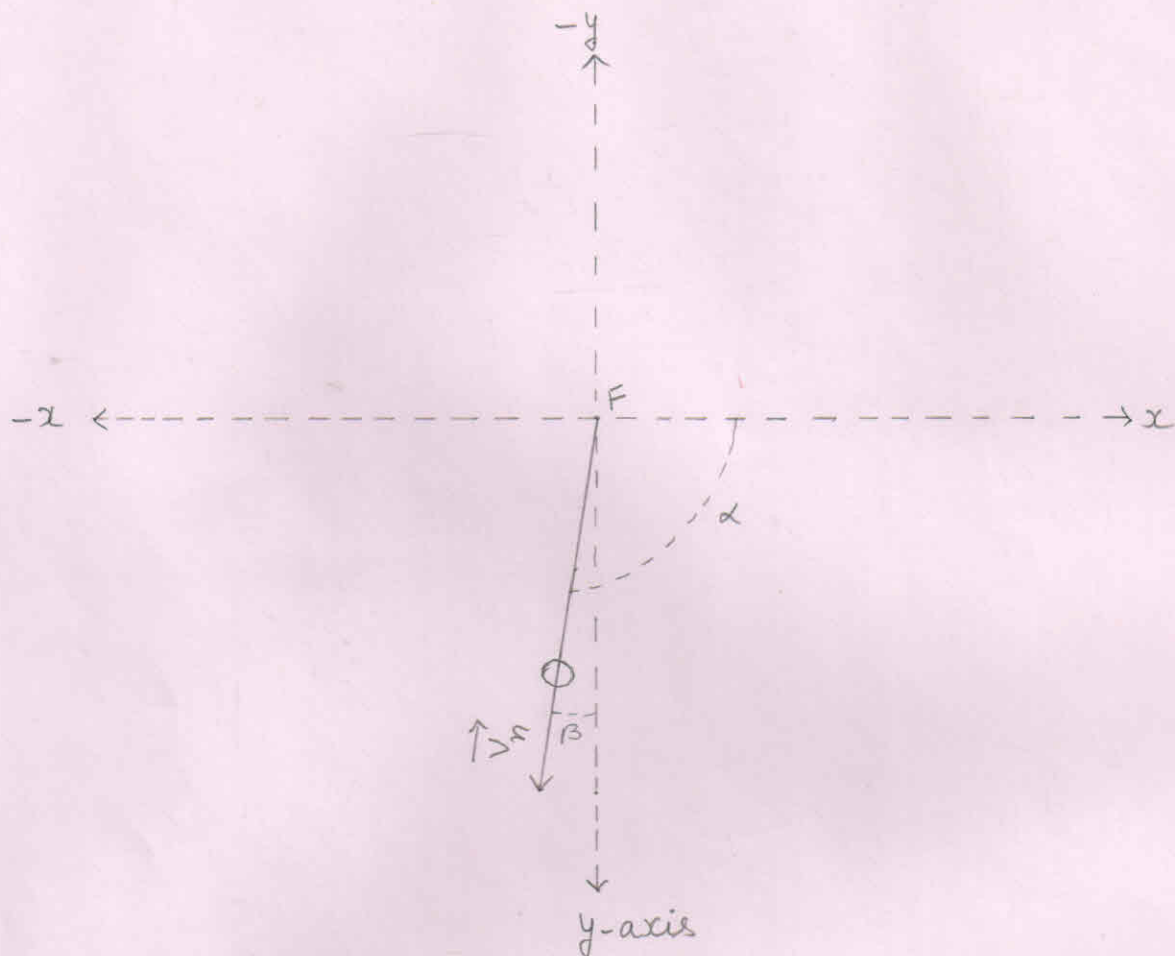
3. With z-axis

$$\cos \gamma = \frac{\vec{v}_z}{v_f}$$

$$\vec{v}_z = 0 \text{ m/s}$$

$$\Rightarrow \cos \gamma = \frac{\vec{v}_z}{v_f} = \frac{0 \text{ m/s}}{v_f} = 0$$

$$\Rightarrow \gamma = 90^\circ$$



$\Rightarrow \vec{V}_n =$ Final velocity of the neutron

\Rightarrow Where,

$$\alpha \approx 98.7 \text{ degree}$$

$$\beta \approx 8.7 \text{ degree}$$

$$\gamma = 90^\circ \text{ degree}$$

Final velocity (v_f) of the helium-4

$$v_f^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_x = 1.1332 \times 10^7 \text{ m/s}$$

$$v_y = 0.4587 \times 10^7 \text{ m/s}$$

$$v_z = 0 \text{ m/s}$$

$$\Rightarrow v_f^2 = (1.1332 \times 10^7)^2 + (0.4587 \times 10^7)^2 + (0)^2 \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f^2 = (1.28414224 \times 10^{14}) + (0.21040569 \times 10^{14}) + (0) \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f^2 = 1.49454793 \times 10^{14} \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_f = 1.2225 \times 10^7 \text{ m/s}$$

Final kinetic energy of the helium-4

$$E = \frac{1}{2} m_{\text{He-4}} v_f^2$$

$$v_f^2 = 1.49454793 \times 10^{14} \text{ m}^2/\text{s}^2$$

$$\Rightarrow E = \frac{1}{2} \times 6.6449 \times 10^{-27} \times 1.49454793 \times 10^{14} \text{ J}$$

$$= 4.9652543877 \times 10^{-13} \text{ J}$$

$$= 3.1032 \text{ MeV}$$

$$\Rightarrow m_{\text{He-4}} v_f^2 = 6.6449 \times 10^{-27} \times 1.49454793 \times 10^{14} \text{ J}$$
$$= 9.9305 \times 10^{-13} \text{ J}$$

Acting forces on the helium-4

$$1. F_y = q v_x B_z \sin\theta$$

$$\vec{v}_x = -1.1332 \times 10^7 \text{ m/s}$$

$$\vec{B}_z = -1 \text{ Tesla}$$

$$q = 2 \times 1.6 \times 10^{-19} \text{ C}$$

$$\sin\theta = \sin 90^\circ = 1$$

$$\Rightarrow F_y = 2 \times 1.6 \times 10^{-19} \times 1.1332 \times 10^7 \times 1 \times 1 \text{ N}$$
$$= 3.6262 \times 10^{-12} \text{ N}$$

From the right hand palm rule, the direction of force \vec{F}_y is according to +y axis. So,

$$\vec{F}_y = 3.6262 \times 10^{-12} \text{ N}$$

$$2. F_z = q v_x B_y \sin\theta$$

$$\vec{B}_y = 1 \text{ Tesla}$$

$$\sin\theta = \sin 90^\circ = 1$$

$$\Rightarrow F_z = 2 \times 1.6 \times 10^{-19} \times 1.1332 \times 10^7 \times 1 \times 1 \text{ N}$$
$$= 3.6262 \times 10^{-12} \text{ N}$$

From the right hand palm rule, the direction of force \vec{F}_z is according to +z axis. So,

$$\vec{F}_z = 3.6262 \times 10^{-12} \text{ N}$$

$$3. F_x = q v_y B_z \sin\theta$$

$$\vec{v}_y = 0.4587 \times 10^7 \text{ m/s}$$

$$\vec{B}_z = -1 \text{ Tesla}$$

$$\sin\theta = \sin 90^\circ = 1$$

$$\Rightarrow F_x = 2 \times 1.6 \times 10^{-19} \times 0.4587 \times 10^7 \times 1 \times 1 \text{ N}$$
$$= 1.4678 \times 10^{-12} \text{ N}$$

From the right hand palm rule, the direction of force \vec{F}_x is according to +x axis. So,

$$\vec{F}_x = 1.4678 \times 10^{-12} \text{ N}$$

4. Resultant force (F_R):

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$$F = 1.4678 \times 10^{-12} \text{ N}$$

$$F_x = F_y = F_z = 3.6262 \times 10^{-12} \text{ N}$$

$$\Rightarrow F_R^2 = F_x^2 + 2F^2$$

$$\Rightarrow F_R^2 = (1.4678 \times 10^{-12})^2 + 2(3.6262 \times 10^{-12})^2 \text{ N}^2$$

$$\Rightarrow F_R^2 = (2.15443684 \times 10^{-24}) + 2(13.14932644 \times 10^{-24}) \text{ N}^2$$

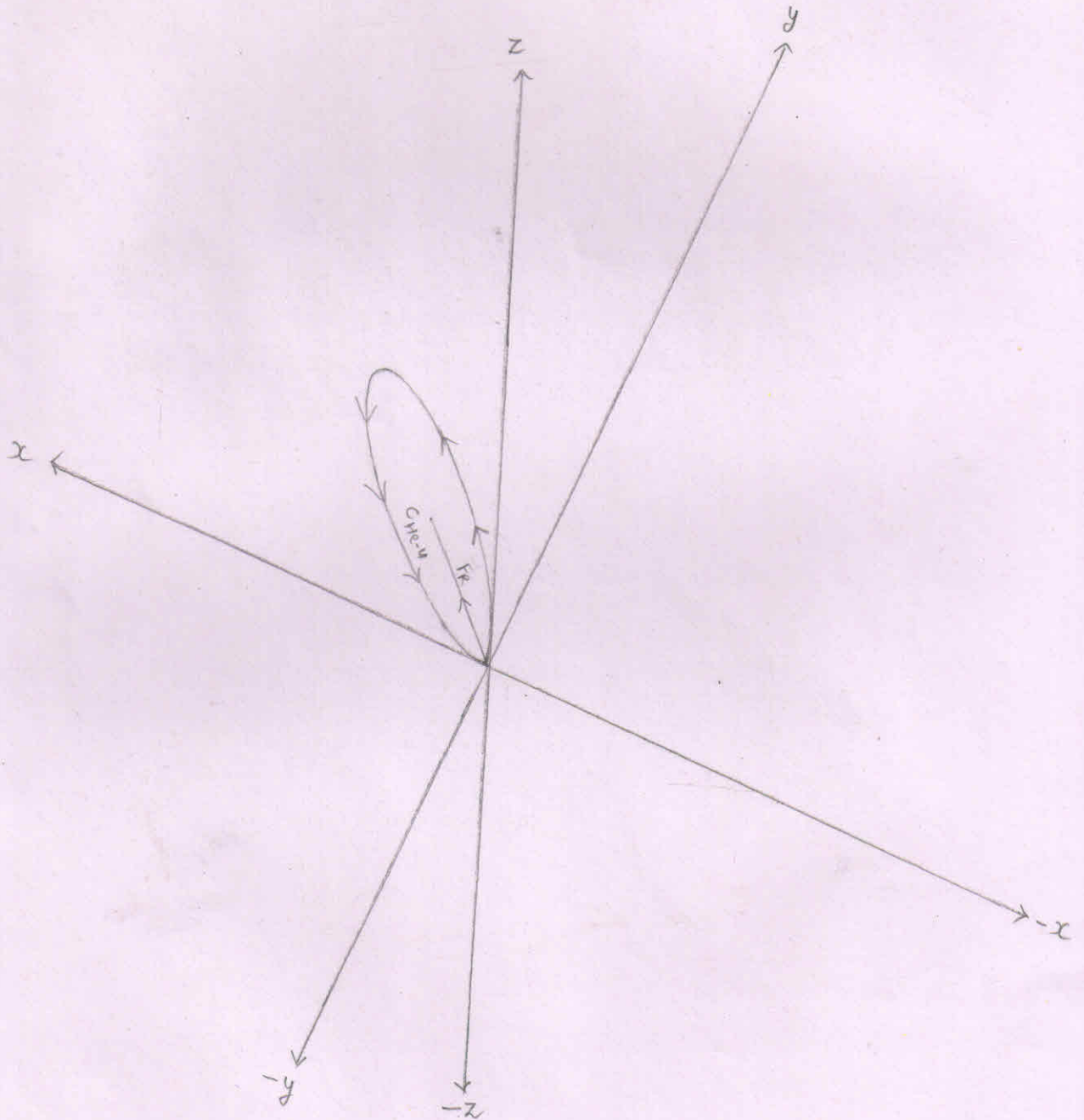
$$\Rightarrow F_R^2 = (2.15443684 \times 10^{-24}) + (26.29865288 \times 10^{-24}) \text{ N}^2$$

$$\Rightarrow F_R^2 = 28.45308972 \times 10^{-24} \text{ N}^2$$

$$\Rightarrow F_R = 5.3341 \times 10^{-12} \text{ N}$$

⇒ The circular orbit to be followed by the helium-4 lies in the I (up) quadrant made up of the positive x axis, positive y axis and positive z axes

⇒ $c_{\text{He-4}}$ = center of the circle to be followed by the helium-4



Angles that make the resultant force (\vec{F}_R)
[acting on the helium-4 nucleus when the
helium-4 nucleus is at point 'F'] with
positive x, y and z-axes :-

1. With x-axis

$$\cos \alpha = \frac{F_R \cos \alpha}{F_R} = \frac{\vec{F}_x}{F_R} = \frac{1.4678 \times 10^{-12} \text{ N}}{5.3341 \times 10^{-12} \text{ N}}$$

$$\Rightarrow \cos \alpha = 0.2751$$

$$\Rightarrow \alpha \approx 74.05$$

$$[\because \cos(74.05) = 0.2747]$$

2. With y-axis

$$\cos \beta = \frac{F_R \cos \beta}{F_R} = \frac{\vec{F}_y}{F_R} = \frac{3.6262 \times 10^{-12} \text{ N}}{5.3341 \times 10^{-12} \text{ N}}$$

$$\Rightarrow \cos \beta = 0.6798$$

$$\Rightarrow \beta \approx 47.2 \text{ degree} \quad [\because \cos(47.2) = 0.6794]$$

3. With z-axis

$$\cos \gamma = \frac{F_R \cos \gamma}{F_R} = \frac{\vec{F}_z}{F_R} = \frac{3.6262 \times 10^{-12} \text{ N}}{5.3341 \times 10^{-12} \text{ N}}$$

$$\Rightarrow \cos \gamma = 0.6798$$

$$\Rightarrow \gamma \approx 47.2 \text{ degree}$$

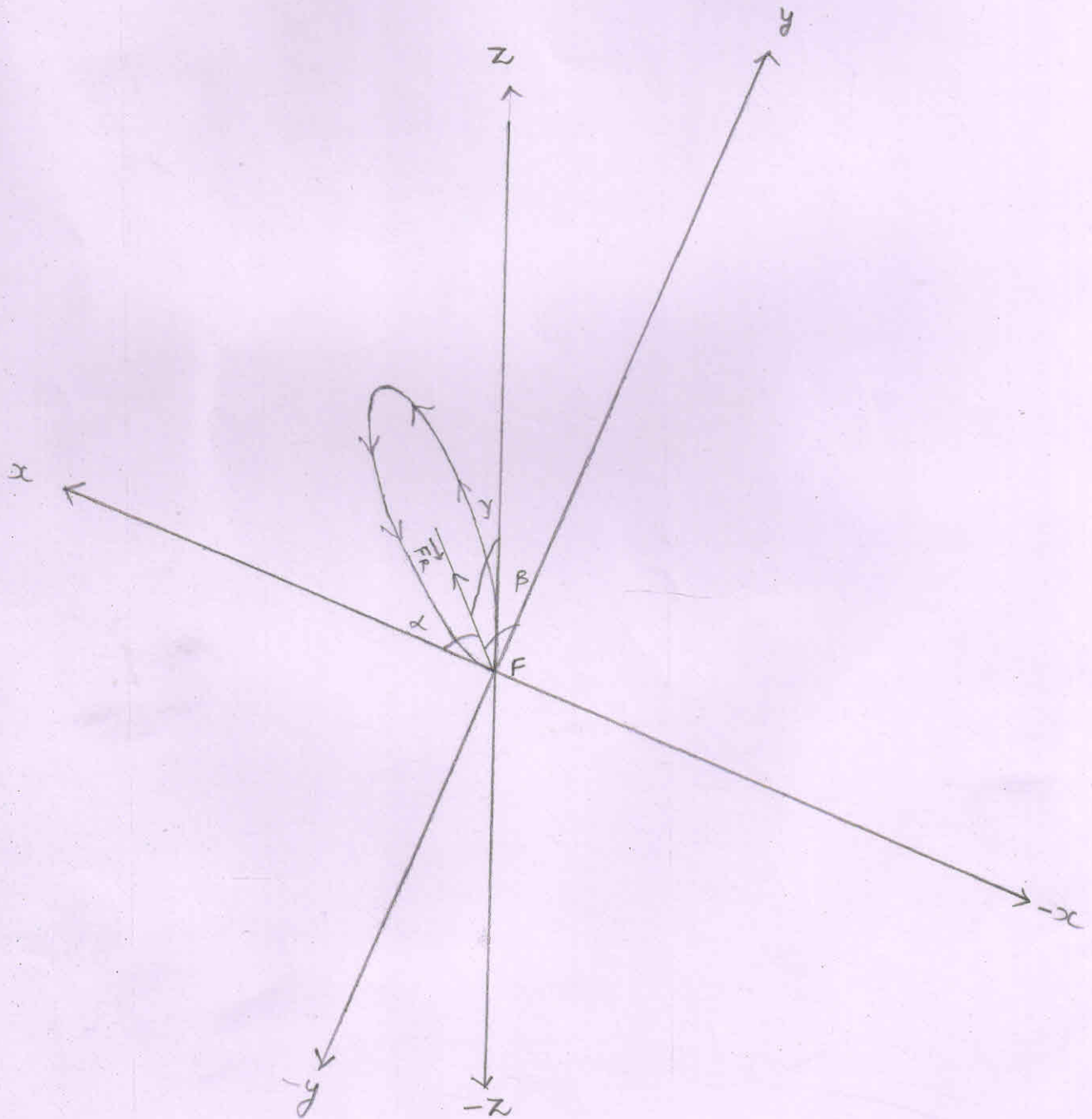
Angles that make the resultant force (\vec{F}_R) at point F with positive x , y and z axes.

Where,

$$\alpha \approx 74.05$$

$$\beta \approx 47.2$$

$$\gamma \approx 47.2$$



5. Radius of the circular orbit followed by the helium-4 :-

$$r = \frac{mv^2}{F_R}$$

$$mv^2 = 9.9305 \times 10^{-13} \text{ J}$$

$$F_R = 5.3341 \times 10^{-12} \text{ N}$$

$$\Rightarrow r = \frac{9.9305 \times 10^{-13}}{5.3341 \times 10^{-12}} \text{ m}$$

$$\Rightarrow r = 1.86170 \times 10^{-1} \text{ m}$$

$$\Rightarrow r = 18.6170 \times 10^{-2} \text{ m}$$

The Cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumference of the circle obtained by the helium-4 nucleus :-

$$1. \cos \alpha = \frac{x_2 - x_1}{d}$$

$$\begin{aligned}d &= 2 \times r \\ &= 2 \times 18.6170 \times 10^{-2} \text{ m} \\ &= 37.234 \times 10^{-2} \text{ m}\end{aligned}$$

$$\cos \alpha = 0.27$$

$$\Rightarrow x_2 - x_1 = d \times \cos \alpha$$

$$\Rightarrow x_2 - x_1 = 37.234 \times 10^{-2} \times 0.27 \text{ m}$$

$$\Rightarrow x_2 - x_1 = 10.0531 \times 10^{-2} \text{ m}$$

$$\Rightarrow x_2 = 10.0531 \times 10^{-2} \text{ m} \quad [\because x_1 = 0]$$

$$2. \cos \beta = \frac{y_2 - y_1}{d}$$

$$\cos \beta = 0.67$$

$$\Rightarrow y_2 - y_1 = d \times \cos \beta$$

$$\Rightarrow y_2 - y_1 = 37.234 \times 10^{-2} \times 0.67 \text{ m}$$

$$\Rightarrow y_2 - y_1 = 24.9467 \times 10^{-2} \text{ m}$$

$$\Rightarrow y_2 = 24.9467 \times 10^{-2} \text{ m} \quad [\because y_1 = 0]$$

$$3. \cos \gamma = \frac{z_2 - z_1}{d}$$

$$\cos \gamma = 0.67$$

$$\Rightarrow z_2 - z_1 = d \times \cos \gamma$$

$$\Rightarrow z_2 - z_1 = 37.234 \times 10^{-2} \times 0.67 \text{ m}$$

$$\Rightarrow z_2 - z_1 = 24.9467 \times 10^{-2} \text{ m}$$

$$\Rightarrow z_2 = 24.9467 \times 10^{-2} \text{ m} \quad [\because z_1 = 0]$$

The Cartesian coordinates of the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ located on the circumference of the circle obtained by the helion-4.

\Rightarrow The line $\overline{P_1P_2}$ is the diameter of the circle.

